Comparing Alternative Factor Models for Forecasting: Case of Turkey

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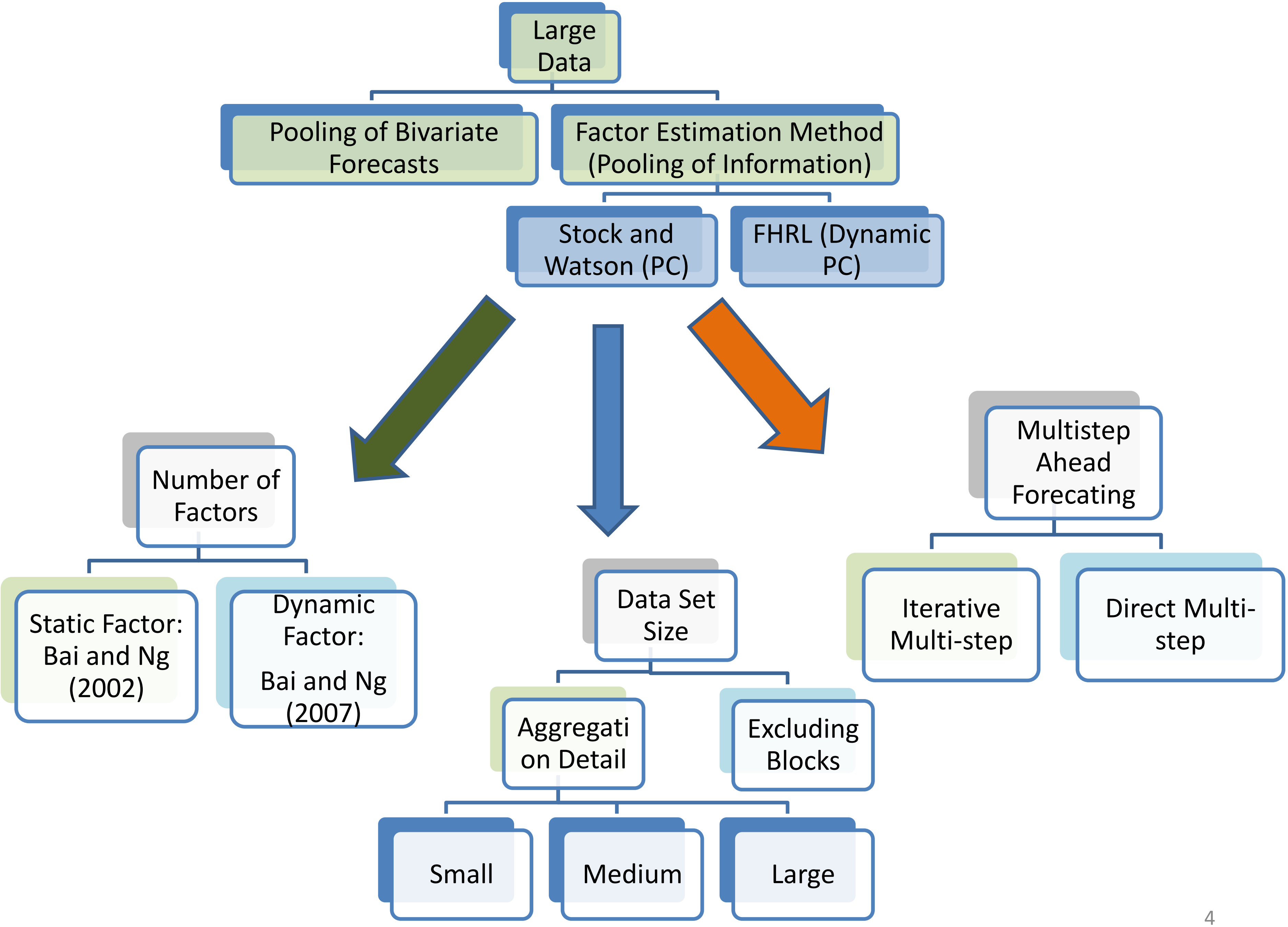
*Views expressed in this presentation are those of the author and do not necessarily represent the official views of the Central Bank of the Republic of Turkey

Factor Model Representation

- $x_{it} = \lambda_{it} f_{1t} + \cdots + \lambda_{ir} f_{rt} + e_{it}$
- $\boldsymbol{X}_{\boldsymbol{it}} = \boldsymbol{\lambda}_{\boldsymbol{i}}' \boldsymbol{F}_{\boldsymbol{t}} + \boldsymbol{e}_{\boldsymbol{it}}$
- X: Observed data
- F: Factors
- $\lambda_i' F_t$: Common component
- $\boldsymbol{e}_{\boldsymbol{it}}$: Idiosyncratic component
- Note that factors, loadings and idiosyncratic components are not-observable.

Issues we need to deal when working with factor models…

- 1. How to get factors?
- 2. How many factors should we use?
- 3. h-period ahead forecast approach: direct or iterative?
- 4. Size and detail of the data set?
- 5. Pooling of bivariate forecasts or factor model forecasts?

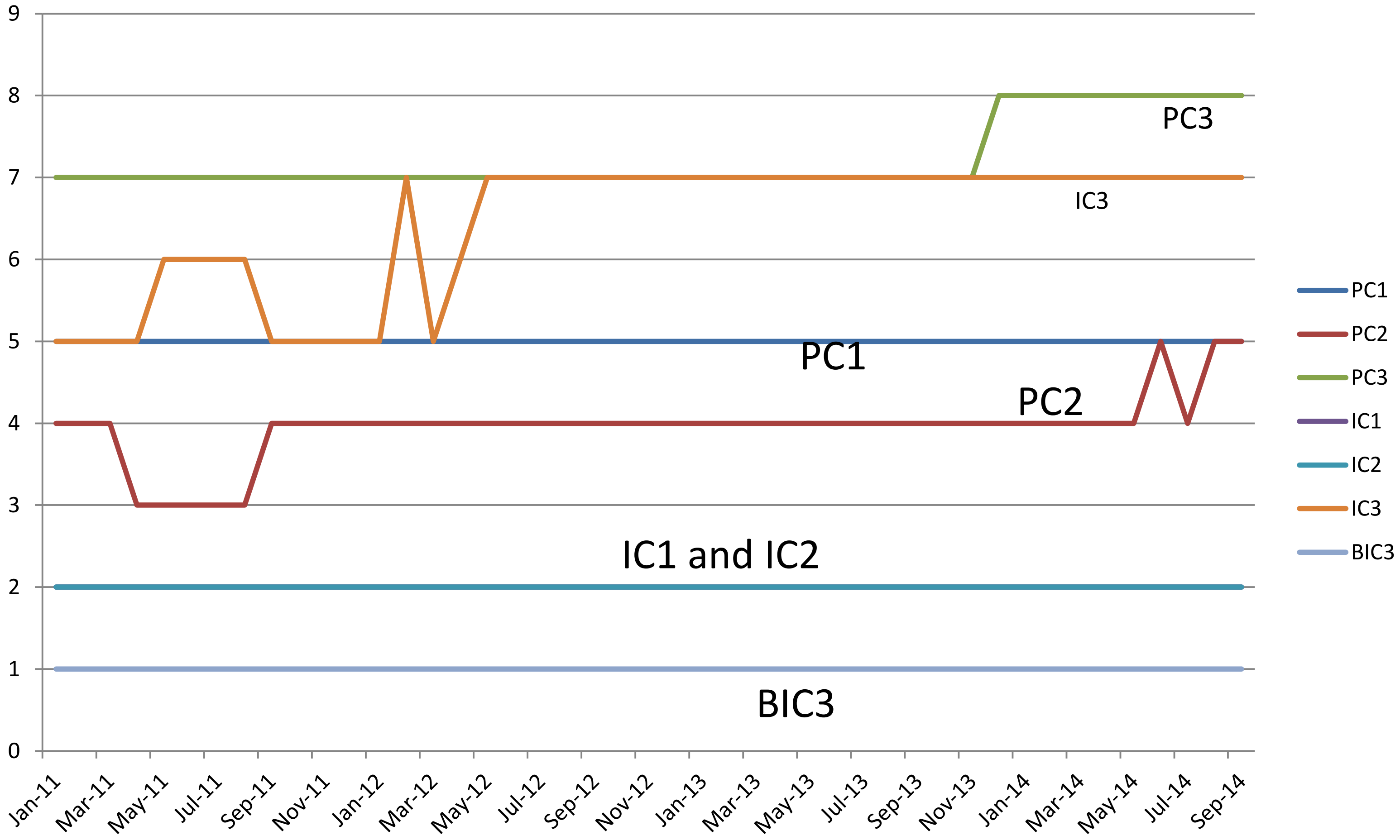

Large Data
Pooling of Bivariate Forecasts
Factor Estimation Method (Pooling of Information)
Stock and Watson (PC)
FHRL (Dynamic PC)
Number of Factors
Static Factor: Bai and Ng (2002)
Dynamic Factor: Bai and Ng (2007)
Data Set Size
Aggregation Detail
Excluding Blocks
Small
Medium
Large
Multistep Ahead Forecating
Iterative Multi-step
Direct Multi-step

Testing the Number of Factors: An Empirical Assessment for a Forecasting Purpose; Barhoumi, Darne and Ferrara (2013), Oxford Bullettin of Economic and Statistics.

Footnote 2 from the paper:

[2] Bai and Ng (2002) proposed two others criteria, $IC_{p2}(k)$ and $IC_{p3}(k)$, where the penalty function is defined as $p_2(n, T) = \left(\frac{(n+T)}{nT}\right) \ln C_{nT}^2$ and $p_3(n, T) = (\ln C_{nT}^2 / C_{nT}^2)$, respectively, with $C_{nT}^2 = \min\{n, T\}$. They also suggested another class of criteria $PC(r)$. Note that the criterion IC_{p1} is the most employed criterion in the forecasting framework. Furthermore, Bai and Ng (2002) showed that the criteria IC_{p1} and IC_{p2} both performed well in their Monte Carlo analysis.

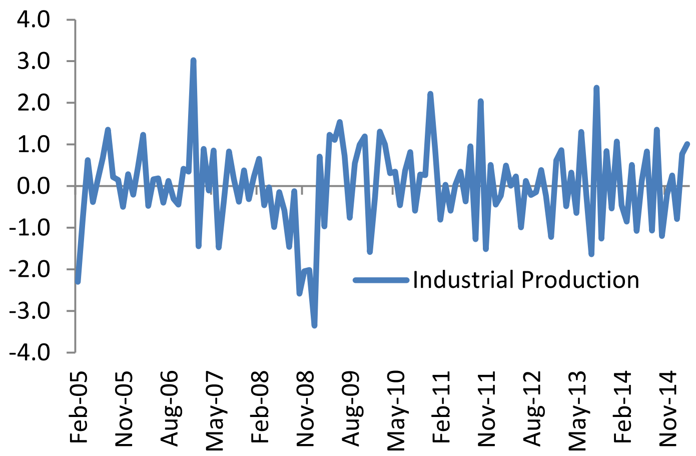
Bai and Ng (2002) criteria applied to a large panel of data for the number of static factors

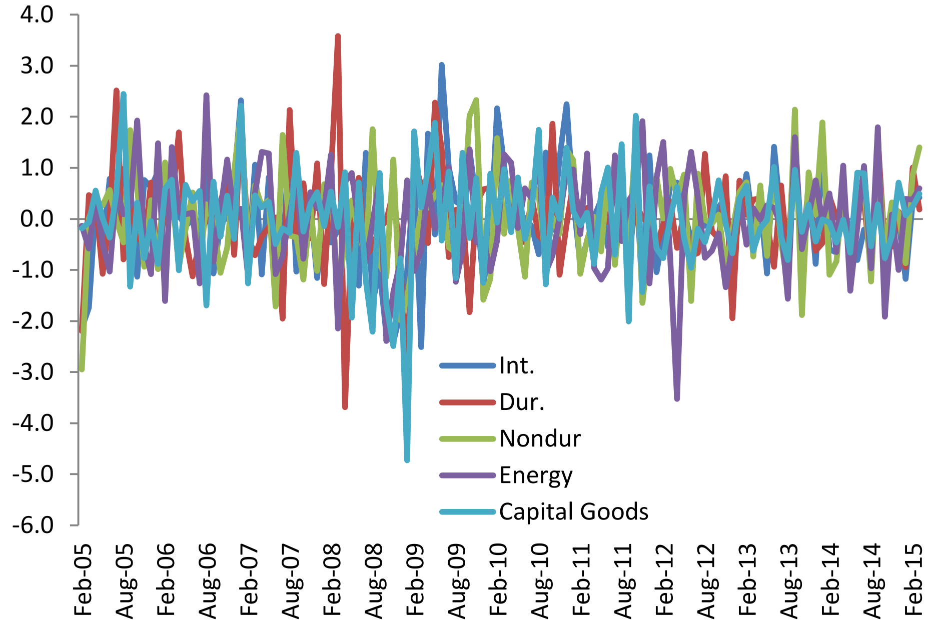


Are Disaggregate Data Useful for Factor Analysis in Forecasting French GDP?, 2010, Barhouimi, Darne and Ferrara, Journal of Forecasting .

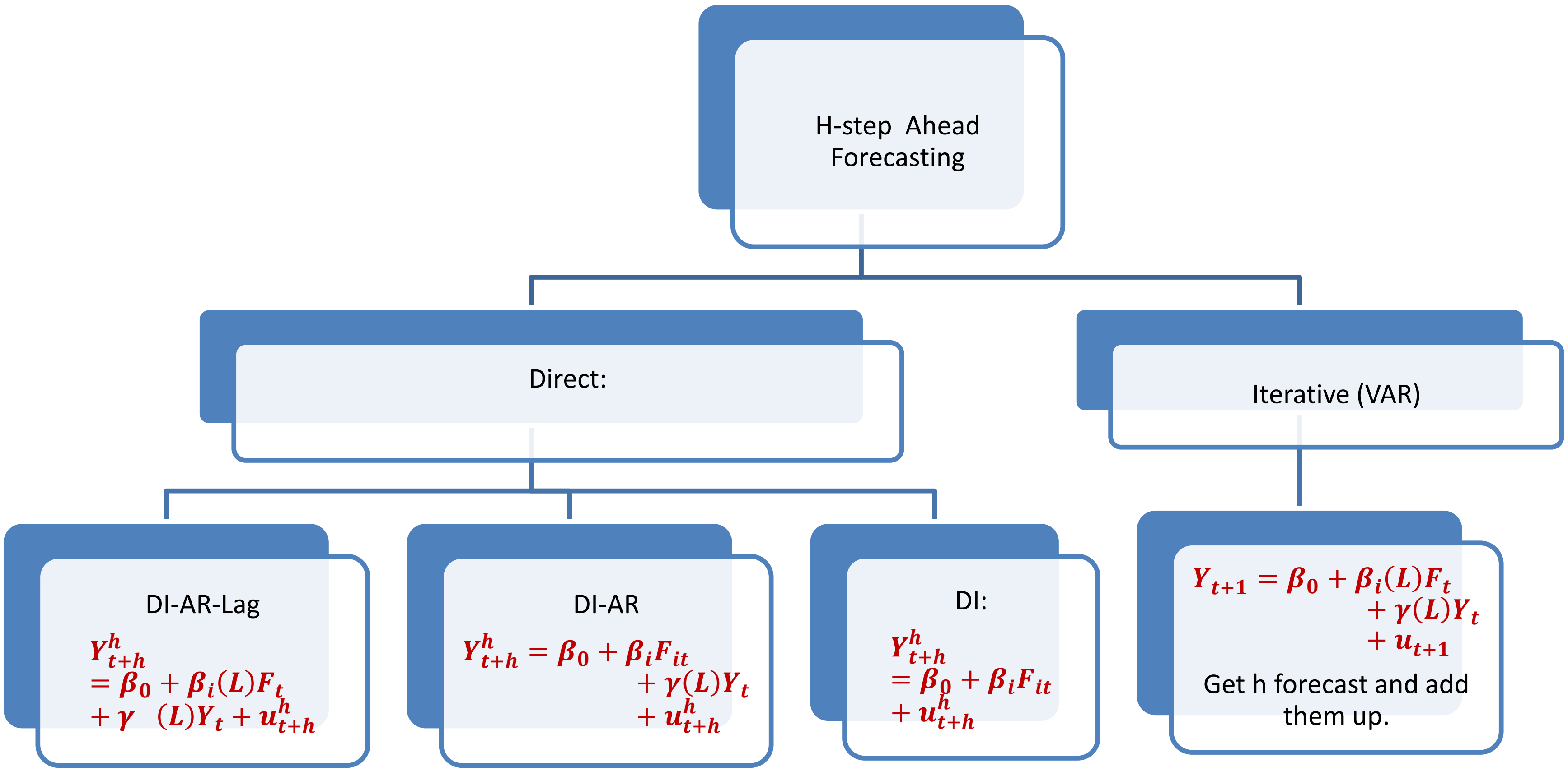
- **Abstract:**
- This paper compares the GDP forecasting performance of alternative factor models based on monthly time series for the French economy.
- These models are based on static and dynamic principal components obtained using time and frequency domain methods. We question whether it is more appropriate to use aggregate or disaggregate data to extract the factors used in forecasting equations.
- **From Conclusion:**
- From this application of the French GDP growth rate, we can conclude that complex dynamic models with a very large database do not necessarily lead to the best forecasting results. Indeed, the simple, **static Stock and Watson (2002a) approach with an aggregated database of 20 series led to comparable forecasting results when using a disaggregated database of 140 series,** especially for nowcasting, where the forecasts are often statistically better.

Month on Month Changes





Forecasting Model Set Up



DMS vs IMS

- A Comparison of Direct and Iterated Multistep AR Methods for Forecasting Macroeconomic Time Series (2006), Marcellino, Stock and Watson, Journal of Econometrics.:The iterated forecasts typically outperform the direct forecasts, particularly if the models can select long lag specifications. The relative performance of the iterated forecasts improves with the forecast horizon.

- Multi-step forecasting in emerging economies: An investigation of the South African GDP, (2009), Chevillon, International Journal of Forecating,:
- To forecast at several, say h, periods into the future, a modeller faces a choice between iterating one-step-ahead forecasts (the IMS technique), or directly modeling the relationship between observations separated by an h-period interval and using it for forecasting (DMS forecasting).
- **It is known that structural breaks, unit-root non-stationarity and residual autocorrelation may improve DMS accuracy in finite samples, all of which occur when modelling the South African GDP over the period 1965-2000.**
- This paper analyzes the forecasting properties of 779 multivariate and univariate models that combine different techniques of robust forecasting. **We find strong evidence supporting the use of DMS and intercept correction, and attribute their superior forecasting performance to their robustness in the presence of breaks.**

Issues we need to deal when working with factor models…

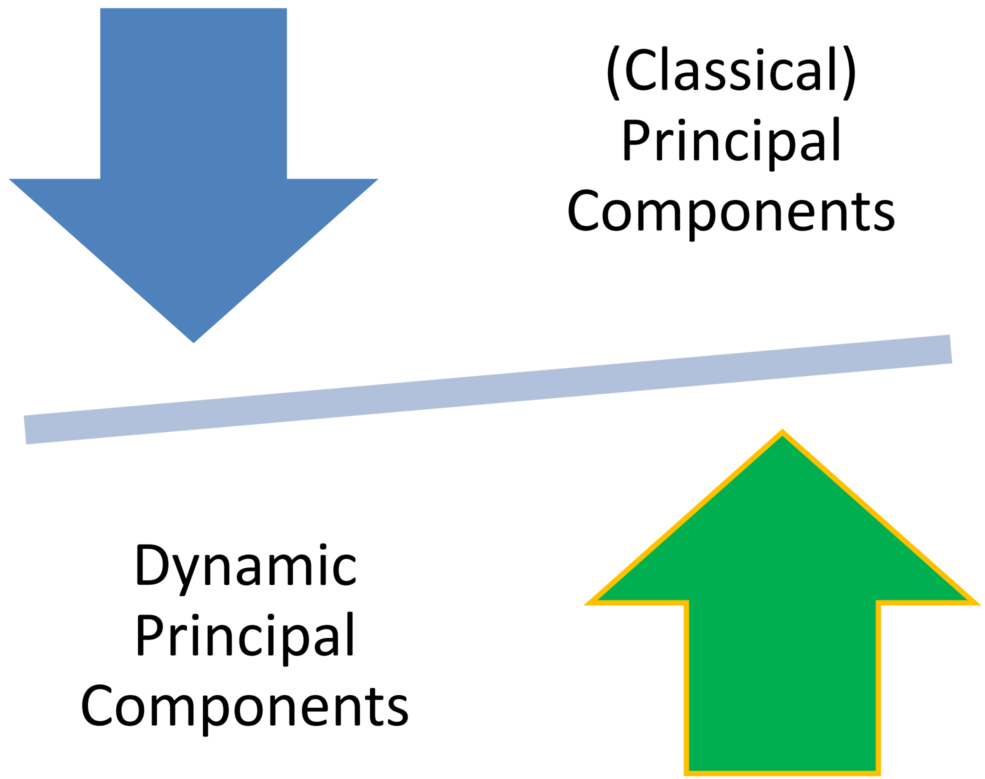
- 1. How to get factors?

Today, we will present how alternative Factor model specification affect forecasting performance?

- 2. How many factors should we use?
- 3. h-period ahead forecast approach: direct or iterative?
- 4. Size and detail of the data set?
- 5. Pooling of bivariate forecasts or factor model forecasts?

Last time,
We analyzed these issues in the context of factors estimated with Principal Components.

Obtaining factors



Obtaining Factors with Principal Components

- Stock and Watson (2002) show that
- $V(\tilde{F},\tilde{\Lambda}) = (NT)^{-1}\sum_i \sum_t (x_{it} - \tilde{\lambda}_i \tilde{F}_t)^2$
- We want to minimize the above loss function which implies that we maximize the part that is explained by the common component.
- $F = X'\hat{\Lambda}$/N solves the above minimization problem.
- $\hat{\Lambda} = eigenvectors\ of\ X'X\ corresponding\ to\ r\ largest\ eigenvalues.$

Obtaining Factors with Dynamic Principal Components a la Forni, Hallin, Lippi, Reichlin (2005)*.

- $X_t = \chi_t + \xi_t = B(L)U_t + \xi_t$
- $B(L) = I + B_1 L + \cdots + B_s L^s$
- *Model can be written in static from as* $X_t = CF_t + \xi_t$, *where*
- $F_t = (U_t', \dots, U_{t-s}')'$ *is a r=q(s+1) dimensional vector of stacked dynamic factors.*
- *In Forni et al. (2005) approach, in the first step we estimate U and in the second step F.*

Presentation of the model is from Schumacher, (2005), Bundesbank Discussion Paper, No 24

Obtaining Factors with Dynamic Principal Components a la Forni, Hallin, Lippi, Reichlin (2005)*.

- Dynamic principal components analysis is used to maximize the variance of the common component.
- We need to solve a dynamic eigenvalue problem of the spectral density matrix of the observed variables.
- Estimated spectrum includes the information on autocorrelations hence provide summary of dynamic relationships.

Presentation of the model is from Schumacher, (2005), Bundesbank Discussion Paper, No 24

Obtaining Factors with Dynamic Principal Components a la Forni, Hallin, Lippi, Reichlin.

- $\hat{\Gamma}(k) = T^{-1} \sum_{t=1}^{T} X_t \, X'_{t-k}$

  $: k - lag\ estimated\ autocovariance\ of\ the\ vector\ of\ time\ series.$

- $\Sigma(\theta_h) = \sum_{k=-M}^{M} \widehat{\Gamma_k} \left(1 - \frac{|k|}{M+1}\right) e^{-ik\theta}$ at frequency $\theta_h = \frac{2\pi h}{2M+1}\ for\ h = 0, \dots, 2M.$
- Spectral density matrix is estimated with a Bartlett window of size M.
- For each frequency dynamic eigenvalues and eigenvectors are computed, and the eigenvectors corresponding to largest q are collected.
- By inverse discrete Fourier transformation, the eigenvectors in the time domain are given by:
- $\hat{P}_j(L) = \sum_{k=-M}^{M} \widehat{P_{j,k}} L^k$ where $\hat{P}_{j,k} = \frac{1}{2M+1} \sum_{h=0}^{2M} \hat{P}_j(\theta_h) e^{ik\theta_h}$
- $\hat{U}_{j,t} = P_j{}'(L) X_t$ are the j-th dynamic principal components

Presentation of the model is from Schumacher, (2005), Bundesbank Discussion Paper, No 24

Obtaining Factors with Dynamic Principal Components a la Forni, Hallin, Lippi, Reichlin.

- Forni et al. (2005) modifies their original factor estimation method which is a two-sided filter to a one-sided filterso that theit can be used in forecasting.
- Variance of the common component is maximized so that the variance of the idiosyncratic component is minimized.
- $\widehat{\sum_{\chi}(\theta)} = \hat{P}(\theta)\Lambda(\theta)\hat{P}^{*}(\theta)$, and $\widehat{\sum_{\zeta}(\theta)} = \widehat{\sum(\theta)}$-$\widehat{\sum_{\chi}(\theta)}$ where $\Lambda(\theta)$ is a qxq diagonal matrix with the largest q dynamic eigenvalues on the main diagonal, and 'P' contain the corresponding eigenvectors.
- $\widehat{\Gamma_{\chi,k}} = \frac{1}{2M+1}\sum_{h=0}^{2M}\widehat{\sum_{\chi}(\theta_h)}\, e^{ik\theta}$, for k=-M,…,M.
- Aim is to find r linear combinations of the time series data $\hat{F}_{j,t} = \hat{Z}_j' X_t$, for j=1,..r that maximize the variance explained by the common factors $\hat{Z}_j{}'\hat{\Gamma}_{\chi,0}\hat{Z}_j$ st $\hat{Z}_j{}'\hat{\Gamma}_{\xi,0}\hat{Z}_i$=1 for i=j and zero otherwise.

Presentation of the model is from Schumacher, (2005), Bundesbank Discussion Paper, No 24

Obtaining Factors with Dynamic Principal Components a la Forni, Hallin, Lippi, Reichlin.

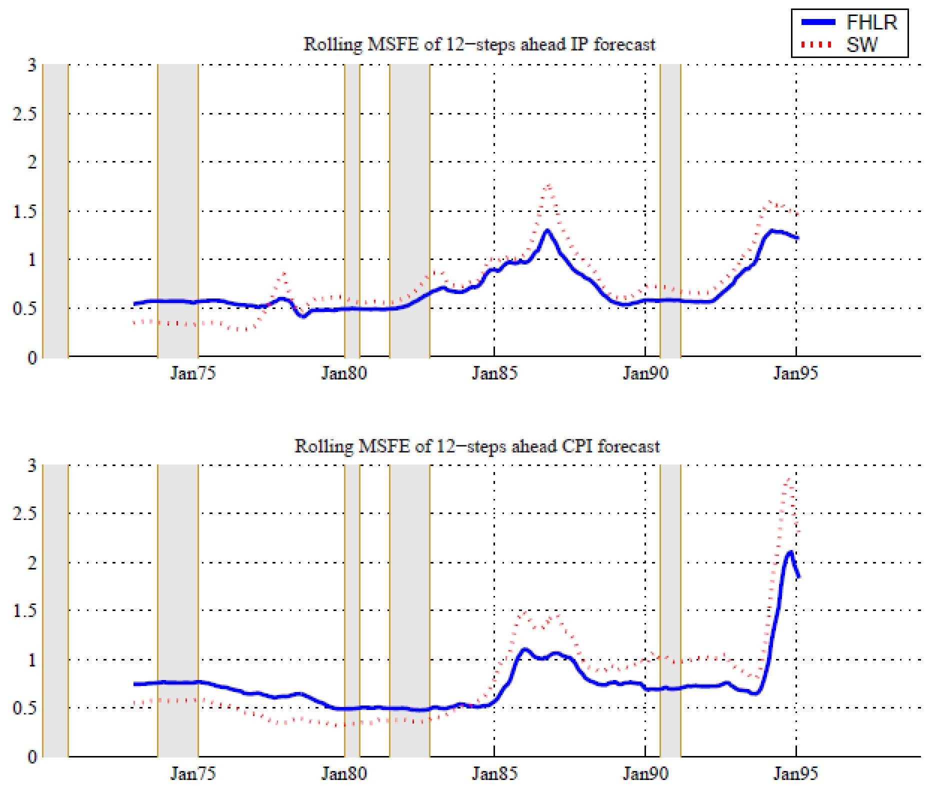
- Finally, we arrive the generalized eigenvalue problem:
- $\widehat{\Gamma}_{\chi,0}\hat{Z}_j = \hat{\mu}_j\widehat{\Gamma}_{\xi,0}\hat{Z}_j,$

  $\mu's\ are\ the\ generalized\ eigenvalue\ and\ Z\ are\ the\ corresponding\ eigenvector.$
- $\widehat{F}_t^{FHLR} = \hat{Z}'X_t$

Presentation of the model is from Schumacher (2007).

Comparing Alternative Factor Models, D'agostino and Giannone, ECB Working Paper,2006, No 680

Figure 1: *Rolling RMSFE of IP and CPI*



Forecasting German GDP Using Alternative Factor Models Based on Large Datasets, 2007, C. Schumacher, Journal of Forecasting.

- Abstract:
- This paper discusses the forecasting performance of alternative factor models based on a large panel of quarterly time series for the German economy.
- One model extracts factors by static principal components analysis; the second model is based on dynamic principal components obtained using frequency domain methods; the third model is based on subspace algorithms for statespace models.
- Out-of-sample forecasts show that the forecast errors of the factor models are on average smaller than the errors of a simple autoregressive benchmark model.
- Among the factor models, the dynamic principal component model and the subspace factor model outperform the static factor model in most cases in terms of mean-squared forecast error.
- **However, the forecast performance depends crucially on the choice of appropriate information criteria for the auxiliary parameters of the models.**
- **In the case of misspecification, rankings of forecast performance can change severely**

Understanding and Comparining Factor Based Forecasts, 2005, Boivin and Ng, International Journal of Central Banking.

- Abstract:
- Forecasting using “diffusion indices” has received a good deal of attention in recent years. The idea is to use the common factors estimated from a large panel of data to help forecast the series of interest. This paper assesses the extent to which the forecasts are influenced by
  - **(i) how the factors are estimated and/or**
  - **(ii) how the forecasts are formulated.**
- We find that for simple data-generating processes and when the dynamic structure of the data is known, no one method stands out to be systematically good or bad. All five methods considered have rather similar properties, though some methods are better in long-horizon forecasts, especially when the number of time series observations is small.
- However, when the dynamic structure is unknown and for more complex dynamics and error structures such as the ones encountered in practice, one method stands out to have smaller forecast errors.
- **This method forecasts the series of interest directly, rather than the common and idiosyncratic components separately, and it leaves the dynamics of the factors unspecified.**
- By imposing fewer constraints, and having to estimate a smaller number of auxiliary parameters, the method appears to be less vulnerable to misspecification, leading to improved forecasts.

‘Small’ Data Set in This Study.
Data are seasonally adjusted and transformed to log-difference or differenced.

1. Industrial Production
2. Export Quantity Index
3. Import Quantity Index
4. Borsa Istanbul-30
5. Business Tendency Survey- Assesment of General Situation
6. Capacity Utilization
7. CNBC-e Consumer Confidence Index
8. Inflation
9. Euro/Dollar Parity
10. Dollar Exchange Rate
11. TL Deposit Interest Rate
12. Dollar Deposit Interest Rate
13. TL Commercial Credit Interest Rate
14. Euro Commercial Credit Interest Rate
15. TL Consumer Credit Interest Rate
16. Benchmark Interest Rate
17. EU-Industrial Production
18. EU Consumer Confidence
19. EU-Business Confidence
20. Commodity Price Index
21. VIX
22. SP 500

Increasing Detail

Small	Medium	Large
• Industrial Production	• Intermediate • Capital • Non-durable • Durable • Energy	• Mining • Food • Beverage • Tobacco • Textile • Apparel • Leather • Wood • Paper • Media • Refined petroleum • Chemical • Pharmaceutical • Rubber • Other Mineral • Basic Metal • Fabricated Metal • Electronic and Optical • Electrical Equipment • Machinery and Equipment • Motor Vehicles • Other Transport • Furniture • Other manufacturing • Repair of mach-eq • Electricity, gas and steam

- For the small set we have 22 series, for medium we have 63 and for the large series we have 167 series.

Recursive Pseudo Out of Sample Forecasting

Estimate in 2005M02-(2012m10-h) to get h step ahead forecast. Get forecasts for h=1 to 12.

Extend sample by one period. Estimate In 2005M02-(2012m11-h) to get h step ahead forecast. Get forecasts for h=1 to 12.

Extend sample by one month

…

Estimate In 2005M02-(2014M09-h) to get h step ahead forecast. Get forecasts for h=1 to 12.

- **Results**

There are many angles and this work can be thought of taking derivative with respect to different factors affecting forecast performance.

- We have **7** options for determining number of static factors, **3** different data size, **2** multi-step ahead forecasting approach where we have **2** alternatives for direct forecasting approach and **2** factor extraction method.
- Aim is to find out whether there is a stable forecasting equation that delivers ‘best’ forecasts.

Moreover, we evaluate models in **2** different episodes since performance may change over time.

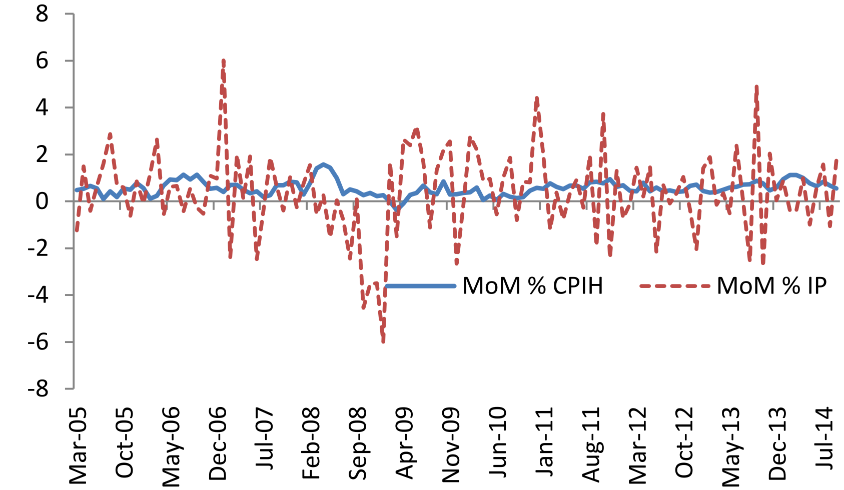
:**Leading Indicators for Euro-Area Inflation and GDP Growth, 2005, Banerjee, Marcellino and Masten, Oxford Bulletin of Economics and Statistics.**

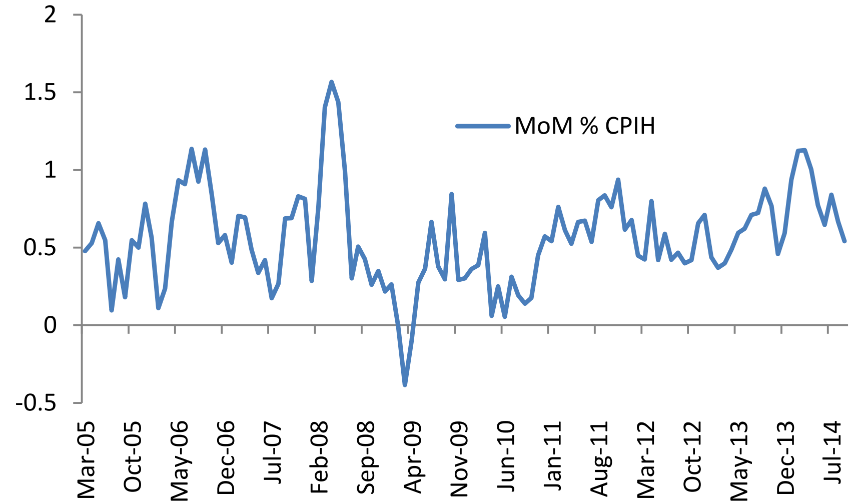
TABLE 1

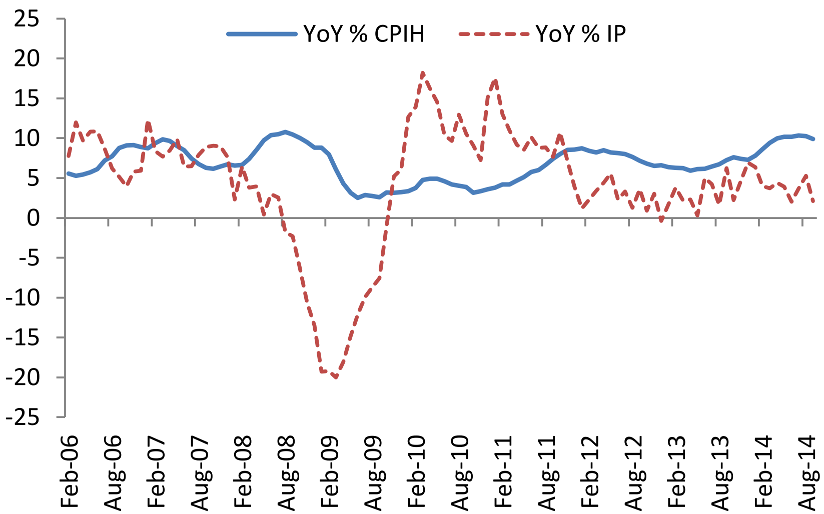
Performance of indicators in forecasting Euro-area inflation (up to four quarters ahead)

	No. of indicators that relative to AR performed		*RMSE-h*		
Estimation period	*Better*	*At least 10% better*	*AR*	*Best indicator*	*Worst indicator*
75:1 84:4	32	29	4.31	1.81 Debt/GDP	4.76 LabProd
75:1 85:4	8	8	1.42	0.60 ComPriceg	5.10 Empl/L
75:1 86:4	22	17	1.65	0.46 TFPg	6.40 IntSpread
75:1 87:4	16	14	0.80	0.25 Empl/L	3.51 NomXR
75:1 88:4	19	9	1.36	0.88 IntSpread	7.78 Surpl/GDP

Target Variables







❖ Since the characteristics of the target variables may be quite different, we analyze two different series, namely Industrial Production and Core Inflation.

Best and Worst Performing Models for IP at h=3 for Two Episodes
RMSE Relative to Simple Benchmark

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
IMS	FHRL	BIC3	M=H=16	Large/9	0.9284
DMS-F	FHRL	BIC3	M=H=16	Large/9	0.9291
DMS	FHRL	BIC3	M=H=16	Large/9	0.9318
IMS	FHRL	IC1	M=H=4	Large/9	0.9324
IMS	FHRL	IC2	M=H=4	Large/9	0.9324
IMS	FHRL	BIC3	M=H=16	Large/9	0.9284

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	PC3	M=H=16	Medium/7	1.122
DMS	FHRL	IC3	M=H=16	Medium/7	1.122
DMS	FHRL	IC3	M=H=4	Large/9	1.128
DMS	FHRL	PC1	M=H=8	Large/9	1.131
AR IMS					1.136
DMS	FHRL	PC1	M=H=16	Large/9	1.145

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
IMS	FHRL	IC2	M=H=16	Medium/7	0.8416
IMS	FHRL	PC1	M=H=16	Medium/2	0.8416
IMS	FHRL	PC2	M=H=16	Medium/2	0.8416
IMS	FHRL	PC3	M=H=16	Medium/2	0.8416
IMS	FHRL	IC1	M=H=16	Medium/2	0.8416
IMS	FHRL	IC2	M=H=16	Medium/2	0.8416

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS	FHRL	IC3	M=H=8	Large/9	1.09
DMS	FHRL	PC1	M=H=4	Small/2	1.10
DMS	FHRL	PC2	M=H=4	Small/2	1.10
DMS	FHRL	PC3	M=H=4	Small/2	1.10
DMS	FHRL	IC1	M=H=4	Small/2	1.10
DMS	FHRL	IC2	M=H=4	Small/2	1.10

Best and Worst Performing Models for IP at h=6 for Two Episodes
RMSE Relative to Simple Benchmark

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	PC3	M=H=16	Medium/7	0.8509
DMS	FHRL	IC3	M=H=16	Medium/7	0.8509
DMS-F	FHRL	BIC3	M=H=16	Large/9	0.8669
IMS	PC	PC1	-	Small/4	0.8679
IMS	PC	PC2	-	Small/4	0.8679
IMS	PC	PC3	-	Small/4	0.8679

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	BIC3	M=H=16	Large/9	0.7812
DMS	FHRL	BIC3	M=H=8	Large/9	0.7844
DMS	FHRL	BIC3	M=H=4	Large/9	0.7926
DMS	PC	BIC3	-	Large/9	0.8558
IMS	FHRL	BIC3	M=H=4	Large/9	0.8967
IMS	FHRL	BIC3	M=H=8	Large/9	0.8971

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS-F	FHRL	IC3	M=H=4	Medium/7	1.087
DMS-F	FHRL	PC3	M=H=16	Large/9	1.103
DMS-F	FHRL	PC3	M=H=8	Large/9	1.117
DMS-F	FHRL	PC3	M=H=4	Large/9	1.120
DMS	FHRL	PC3	M=H=4	Large/9	1.129
DMS-F	PC	PC3	-	Large/9	1.162

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS	FHRL	PC1	M=H=8	Large/9	1.5768
DMS	PC	PC3	-	Large/9	1.7216
DMS	PC	IC3	-	Large/9	1.7237
DMS	PC	PC3	-	Medium/7	1.7507
DMS	PC	IC3	-	Medium/7	1.7507
DMS	FHRL	PC3	M=H=16	Medium/7	1.8966

Best and Worst Performing Models for IP at h=9 for Two Episodes
RMSE Relative to Simple Benchmark

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	PC	PC3	-	Medium/7	0.6201
DMS	PC	IC3	-	Medium/7	0.6201
IMS	FHRL	BIC3	M=H=16	Large/9	0.7463
IMS	FHRL	BIC3	M=H=8	Large/9	0.7547
DMS	PC	PC1	-	Medium/7	0.7586
DMS	PC	PC2	-	Medium/7	0.7591

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	BIC3	M=H=16	Large/9	0.8619
DMS	FHRL	BIC3	M=H=8	Large/9	0.8647
DMS	FHRL	BIC3	M=H=4	Large/9	0.8733
DMS	FHRL	IC2	M=H=4	Small/4	0.8797
DMS	PC	BIC3	-	Large/9	0.9065
DMS	FHRL	BIC3	M=H=4	Medium/2	0.9127

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	IC3	M=H=16	Large/9	1.113
DMS	FHRL	PC3	M=H=16	Large/9	1.146
DMS-F	FHRL	PC3	M=H=4	Large/9	1.199
DMS	FHRL	IC3	M=H=8	Large/9	1.223
DMS	FHRL	PC1	M=H=8	Large/9	1.231
DMS-F	FHRL	PC3	M=H=16	Large/9	1.233

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS	FHRL	PC1	M=H=16	Large/9	2.4201
DMS	PC	PC3	-	Large/9	2.5829
DMS	PC	IC3	-	Large/9	2.5911
DMS	FHRL	PC3	M=H=8	Medium/7	2.7192
DMS	FHRL	IC3	M=H=8	Medium/7	2.7192
DMS	FHRL	PC3	M=H=4	Large/9	2.9385

Best and Worst Performing Models for IP at h=12 for Two Episodes RMSE Relative to Simple Benchmark

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	PC	PC3	-	Medium/7	0.6304
DMS	PC	IC3	-	Medium/7	0.6304
DMS-F	PC	PC1	-	Small/4	0.6977
DMS-F	PC	PC2	-	Small/4	0.6977
DMS-F	PC	PC3	-	Small/4	0.6977
DMS-F	PC	IC1	-	Small/4	0.6977

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	BIC3	M=H=4	Medium/7	0.7418
DMS	FHRL	BIC3	M=H=8	Medium/7	0.7422
DMS	FHRL	BIC3	M=H=16	Medium/7	0.7452
DMS	PC	BIC3	-	Medium/7	0.7590
DMS	PC	IC1	-	Large/9	0.7829
DMS	PC	IC2	-	Large/9	0.7829

Multistep Ahead Forecassting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	IC3	M=H=16	Large/9	1.088
DMS	FHRL	PC3	M=H=16	Large/9	1.114
DMS	FHRL	PC3	M=H=8	Large/9	1.150
DMS-F	FHRL	PC3	M=H=4	Large/9	1.155
DMS-F	FHRL	PC3	M=H=8	Large/9	1.172
DMS-F	FHRL	PC3	M=H=16	Large/9	1.180

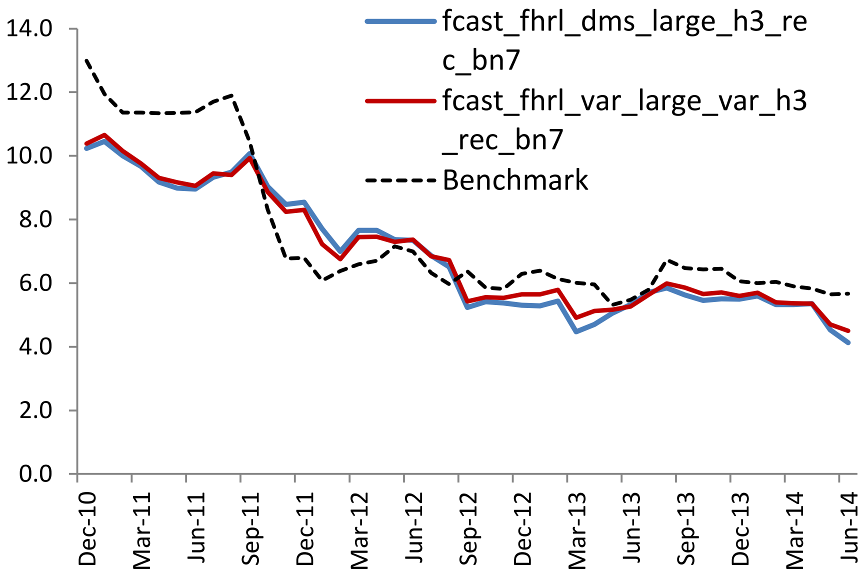
Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS	FHRL	PC3	M=H=8	Medium/7	3.2998
DMS	FHRL	IC3	M=H=8	Medium/7	3.2998
DMS	PC	PC1	-	Large/9	3.3313
DMS	PC	PC3	-	Large/9	3.3424
DMS	PC	IC3	-	Large/9	3.3556
DMS	FHRL	PC3	M=H=4	Large/9	3.4934

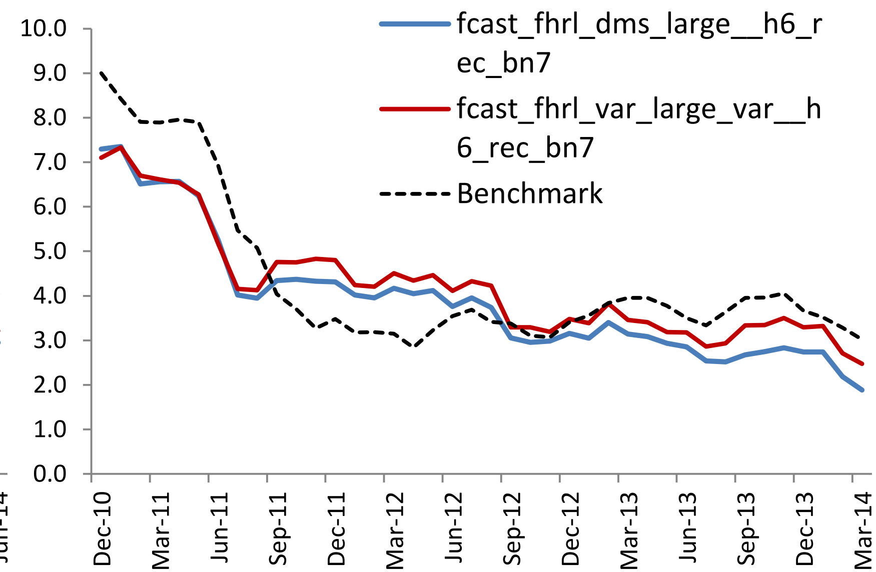
To sum up…

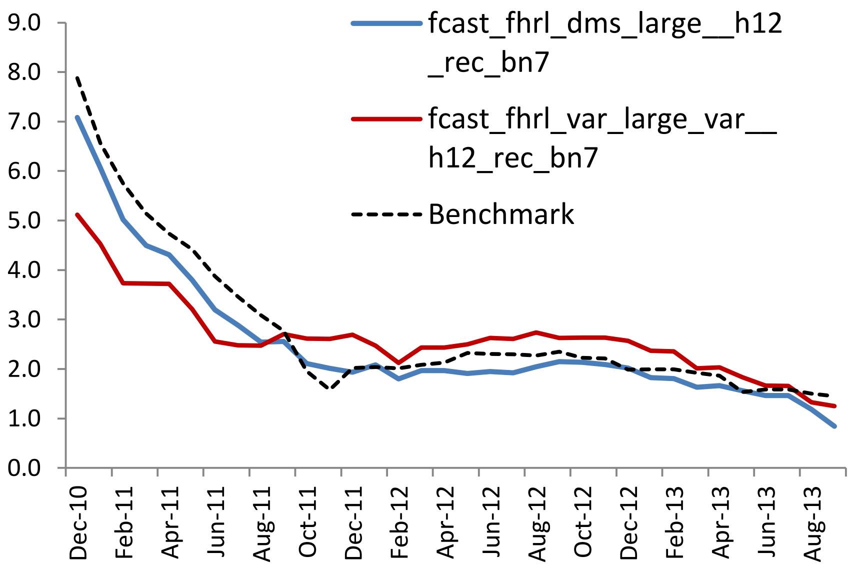
- Model specification changes depending on forecast horizon and forecast evaluation sample.
- Number of factors seem to play a significant role on the forecast performance while the multi-step ahead forecasting and forecast estimation methods seem to matter less.
- Since we can show only selected top items in the tables for a limited time period, we will analyze the results with graphs.
- To have a better idea about the stability of the results over time, we will present relative RMSEs with 12 month rolling window (e.g. Jan 2010-Dec 2010, Feb 2010-Jan 2011,…)

Multi-Step Ahead Forecasting Approach

12 Month-Moving RMSEs for Different Specifications, DMS vs VAR given BIC3 and Large data set

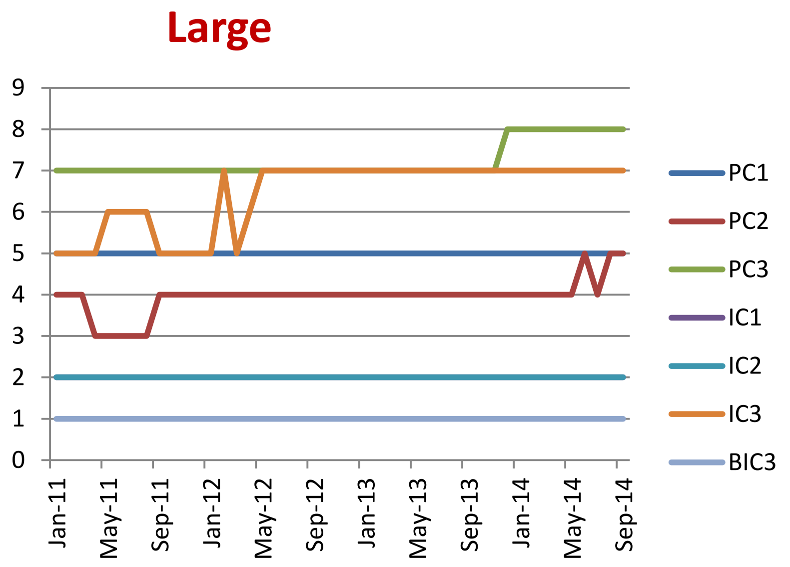


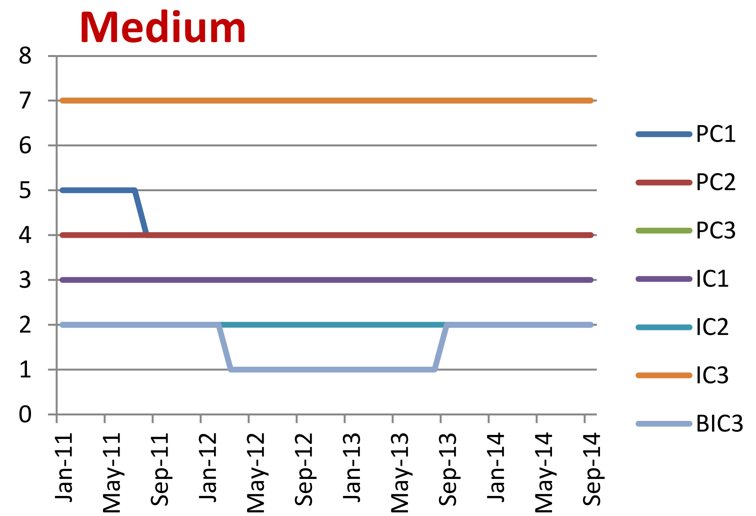


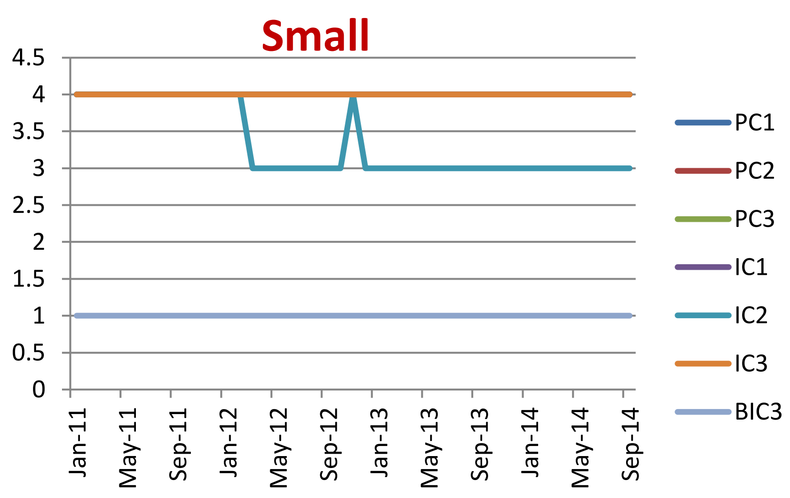


❖ DMS improves over IMS at longer horizons.

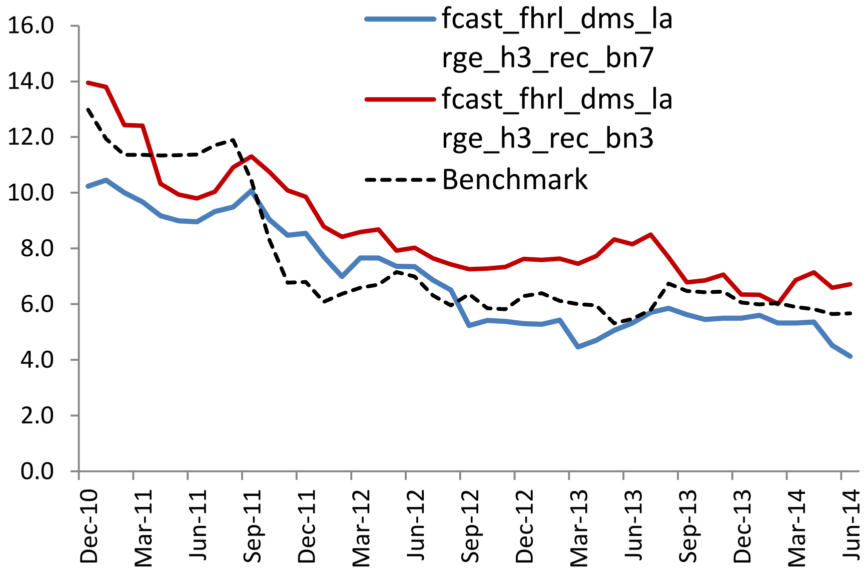
- **Factor Extraction Methods and Number of Factors**

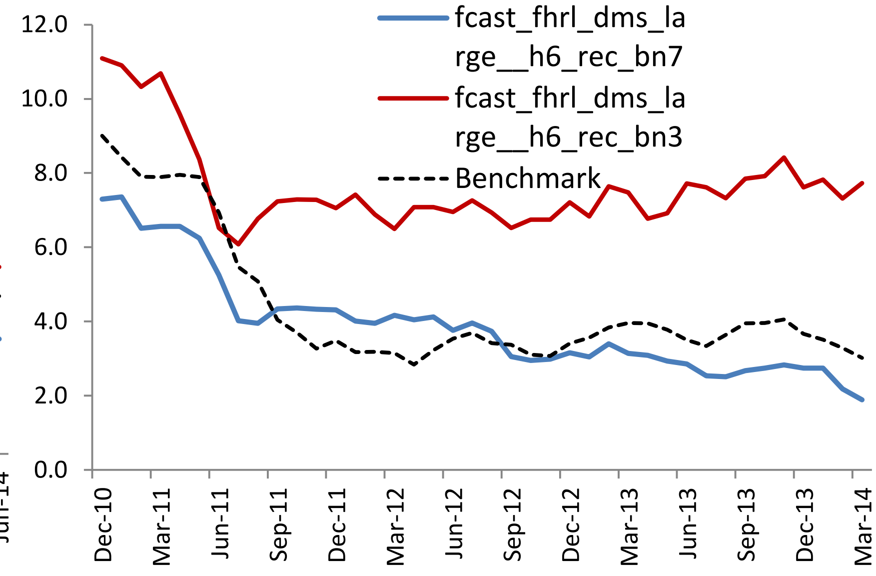


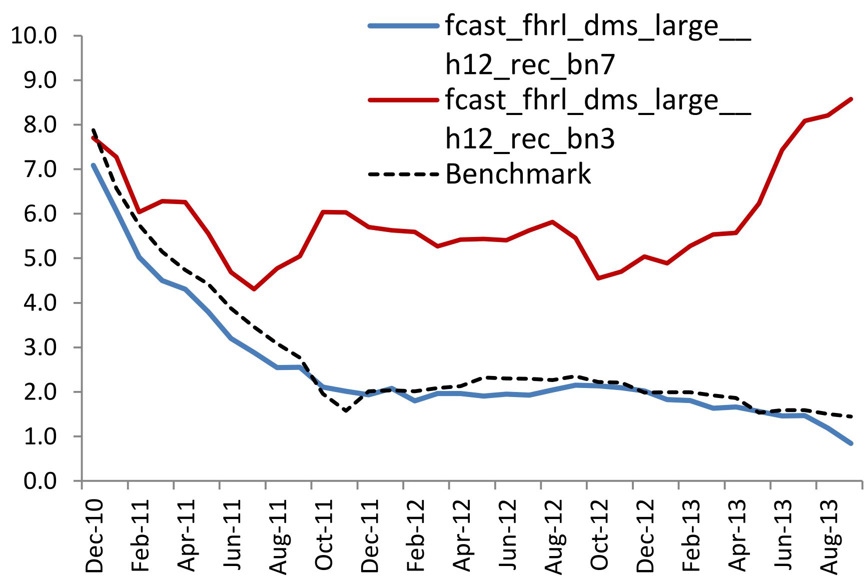




Rolling 12 Month RMSEs for Different Specifications, BIC3 vs PC3 given Large data set and DMS







❖ In the case of direct forecasting, using a large number of factors worsens the forecast performance.

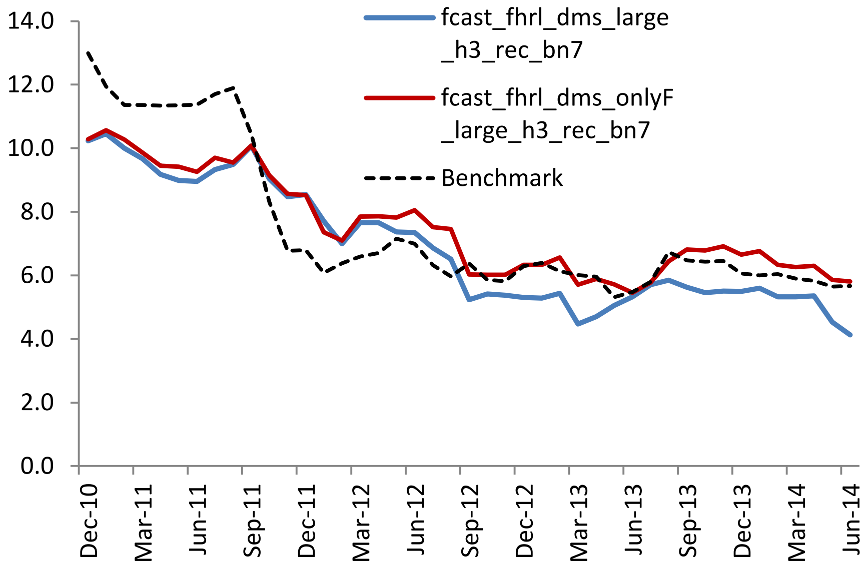
Direct Multi-Step Ahead Forecasting Equation Set-Ups

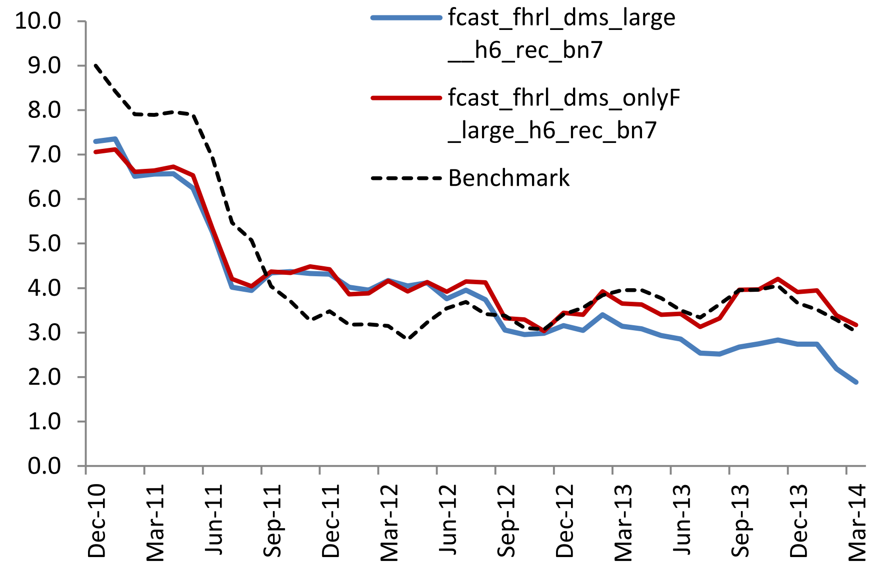
DI-AR-Lag

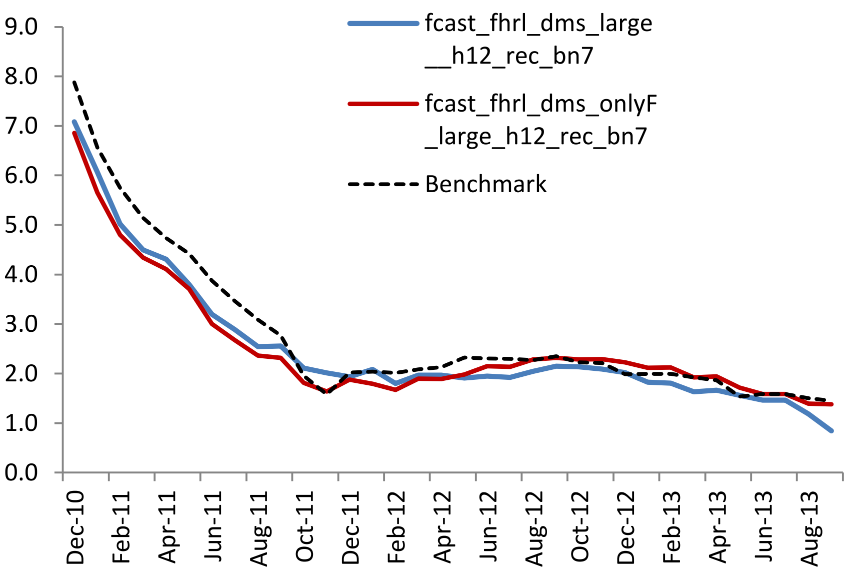
$$Y_{t+h}^{h} = \beta_0 + \beta_i(L)F_t + \gamma \quad (L)Y_t + u_{t+h}^{h}$$

DI: $Y_{t+h}^{h} = \beta_0 + \beta_i F_{it} + u_{t+h}^{h}$

Rolling 12 Month RMSEs for Different Specifications, AR-DI-Lag vs DI given BIC3, FHRL and Large data set

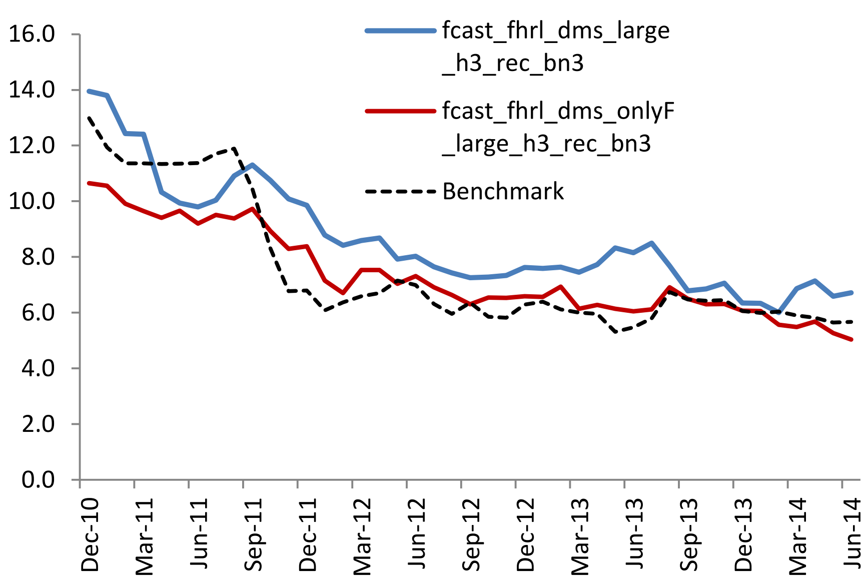


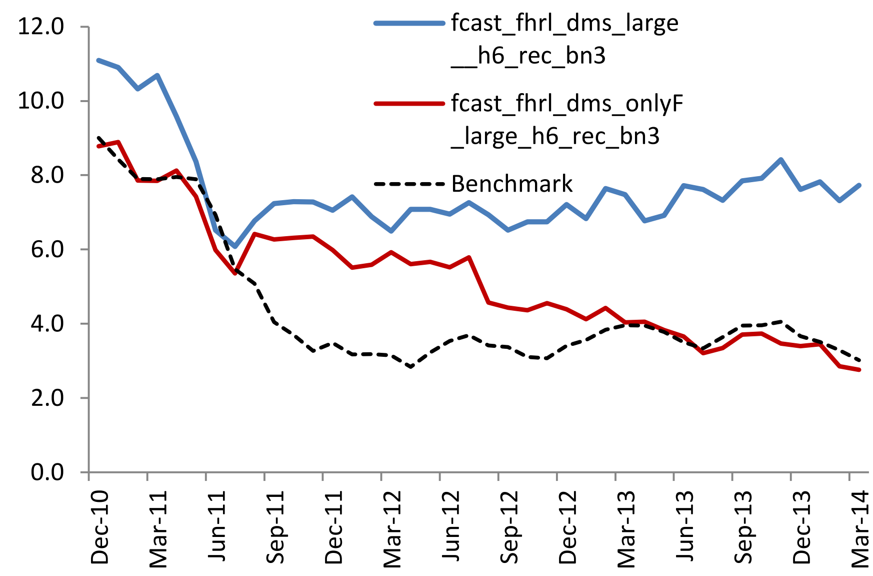


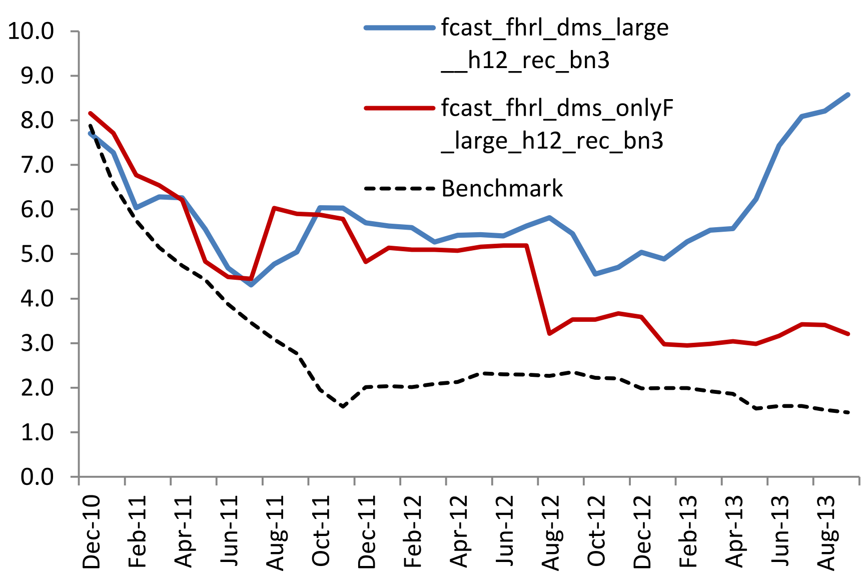


❖ In the case of using few factors, DI-AR-LAG gives relatively more successful forecasts in the recent period.

Rolling 12 Month RMSEs for Different Specifications, AR-DI-Lag vs DI given PC3, FHRL and Large data set

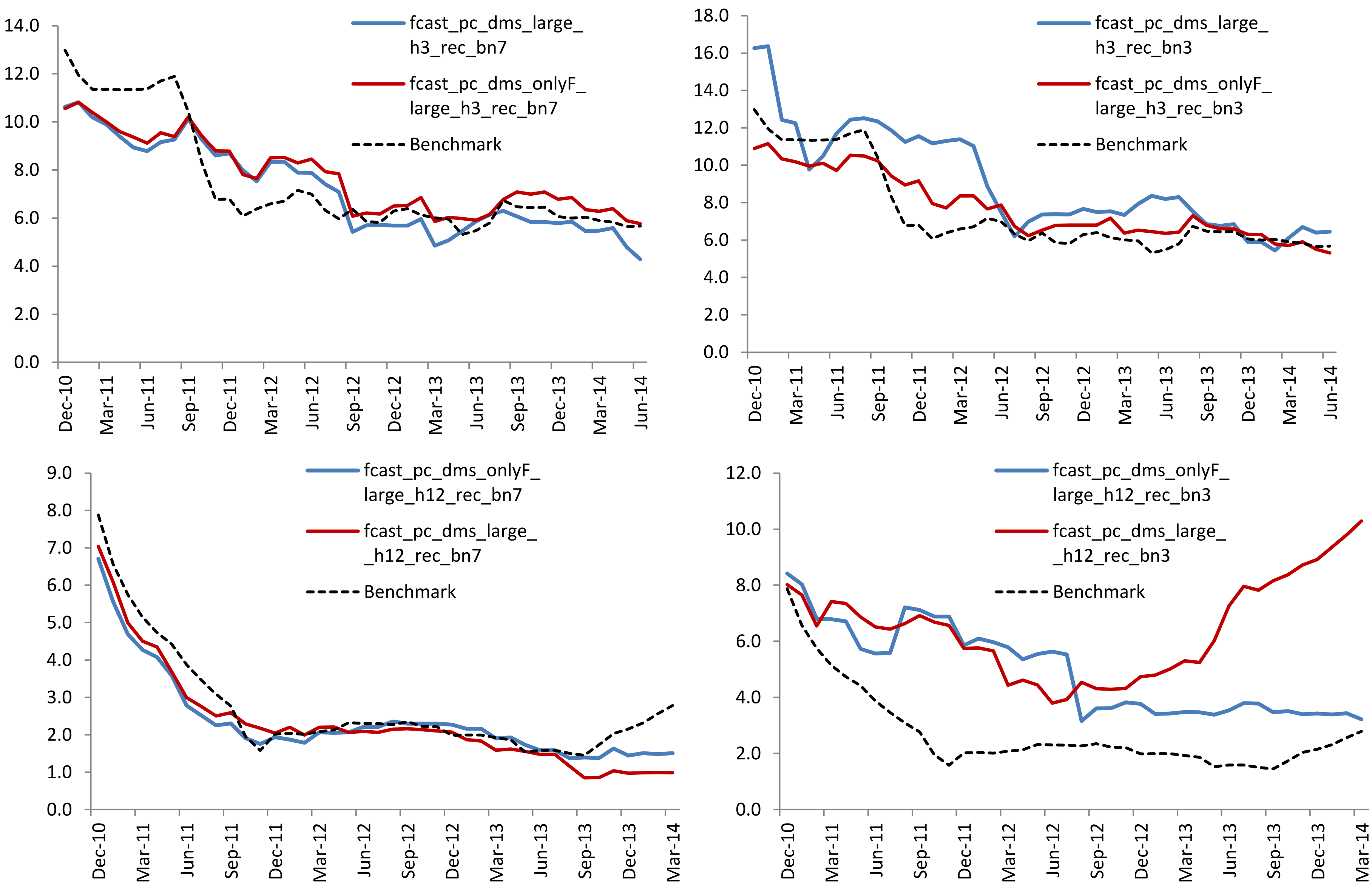






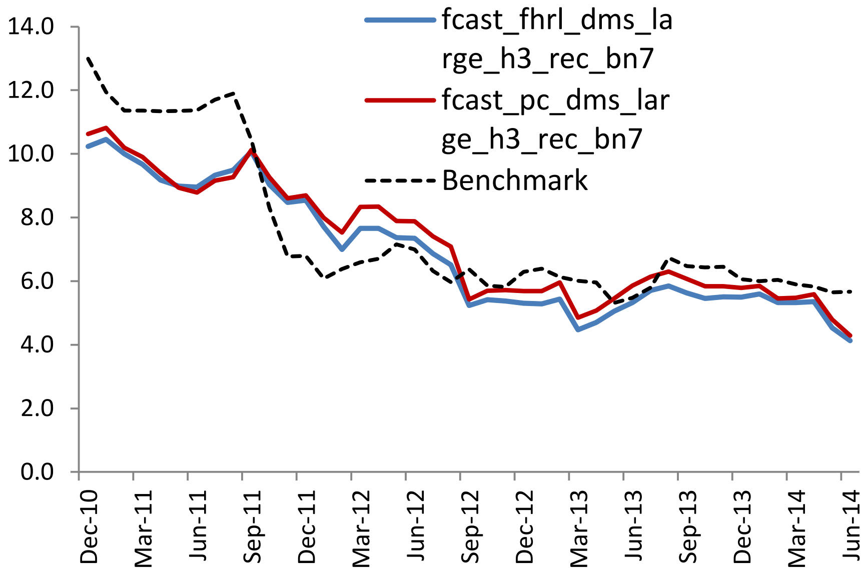
❖ In the case of using a high number of factors, DI gives relatively more successful forecasts.

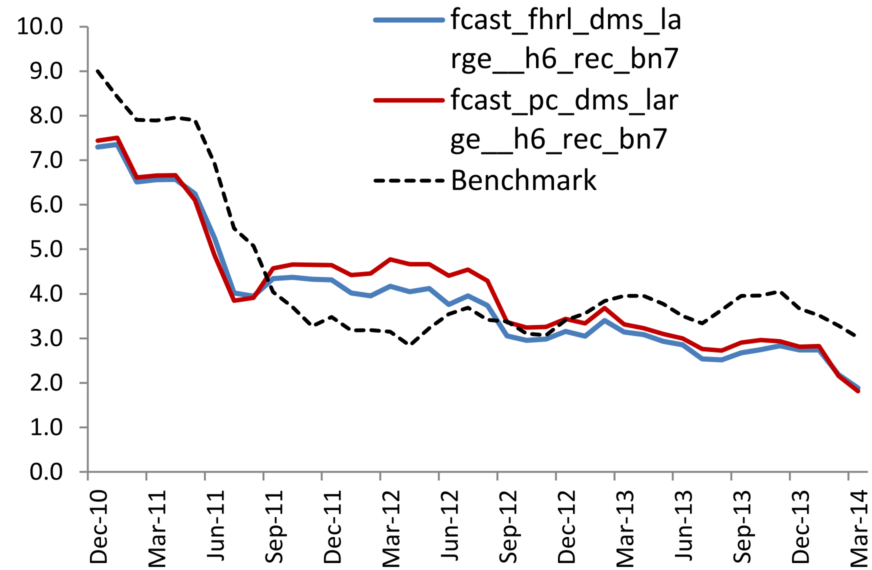
Rolling 12 Month RMSEs for Different Specifications, AR-DI-Lag vs DI for BIC3 vs PC3, given PC and Large data set

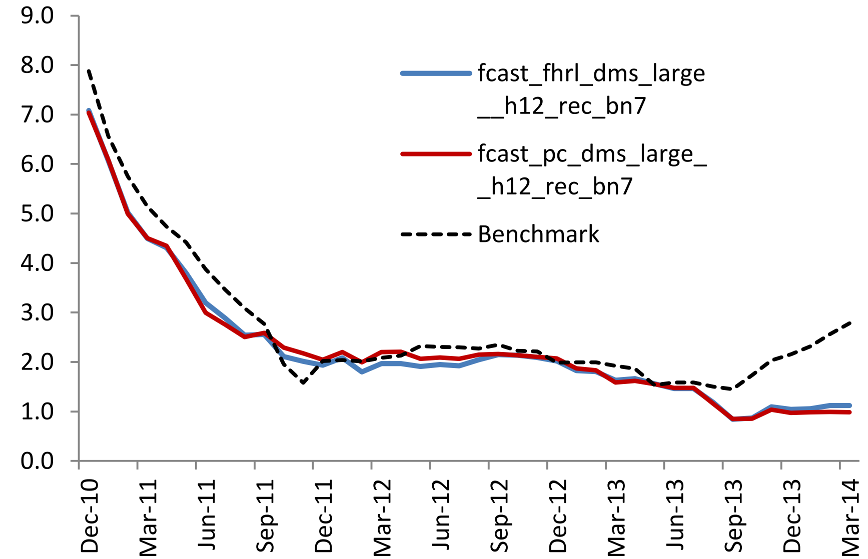


FHRL vs PC

Rolling 12 Month RMSEs for Different Specifications, PC vs FHLR given BIC3 and Large data set

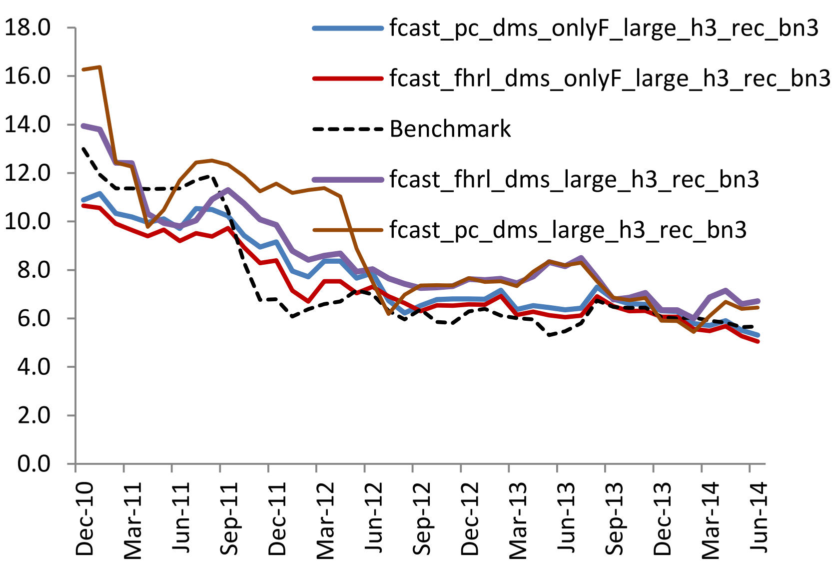


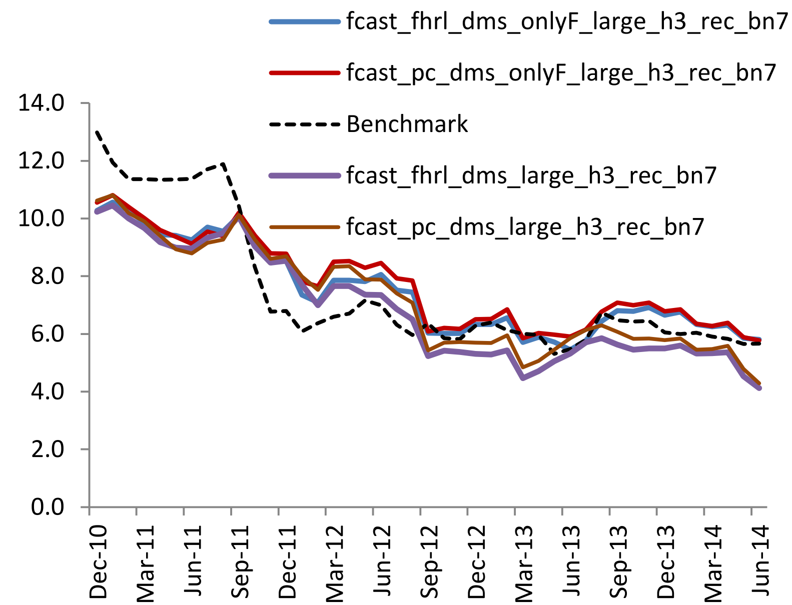


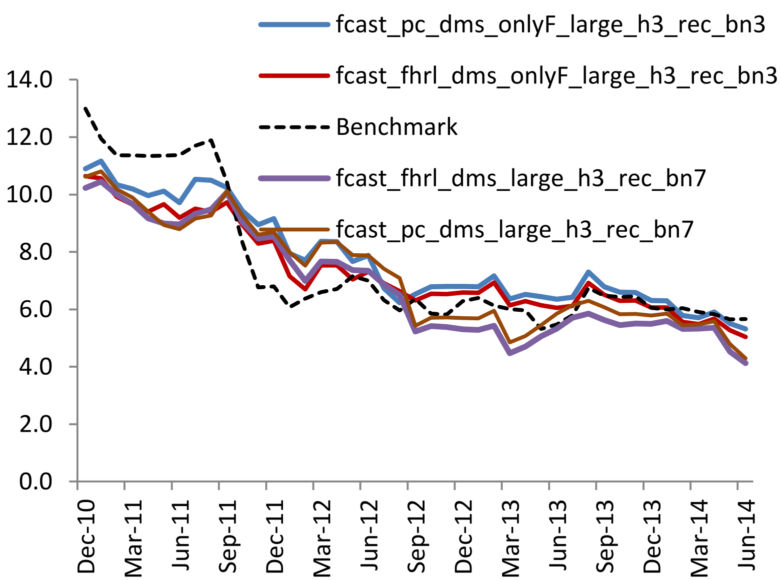


- When we use BIC3 which tends to deliver few factors, there is limited gain in using FHLR.

Rolling 12 Month RMSEs for Different Specifications, PC vs FHLR for PC3 vs BIC3 and DI-AR-Lag vs DI given h3 and Large data set

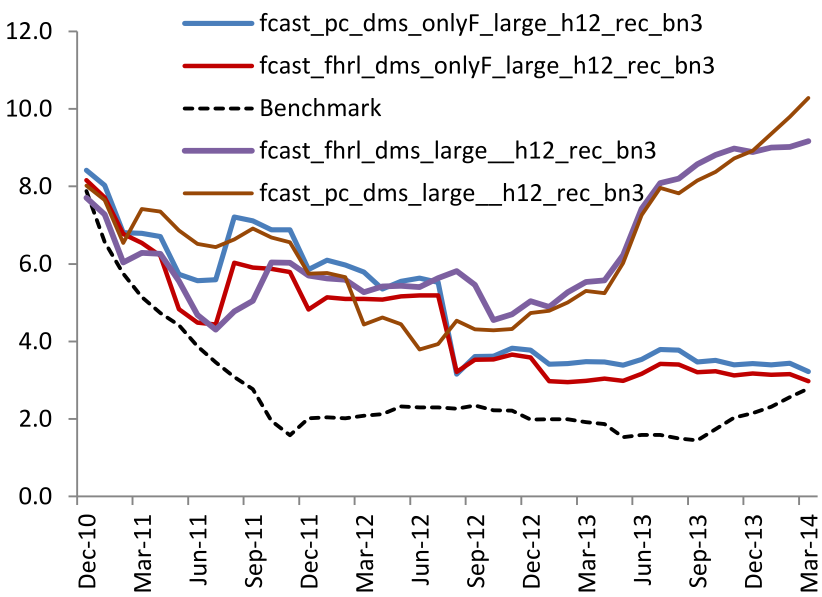


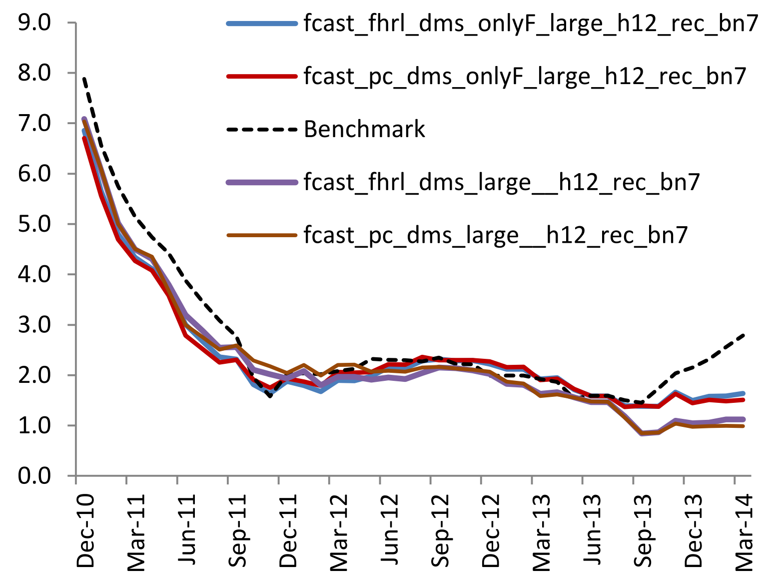




- When we consider h=3, for the criterion that delivers a large number of factors (i.e. PC3), **DI-AR-Lag** model performs relatively worse.
- On the other hand, for BIC3 where we work with a parsimonious model, **DI-AR-Lag** model performs relatively better.

Rolling 12 Month RMSEs for Different Specifications, PC vs FHLR for PC3 vs BIC3 given h12 and Large data set





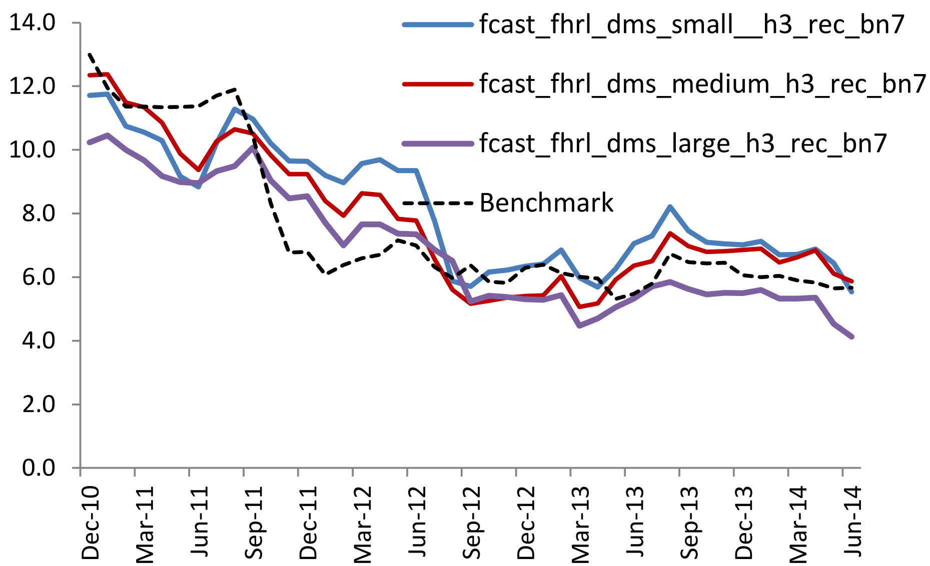
- When we consider h=12, for the criterion that delivers a large number of factors (i.e. PC3),

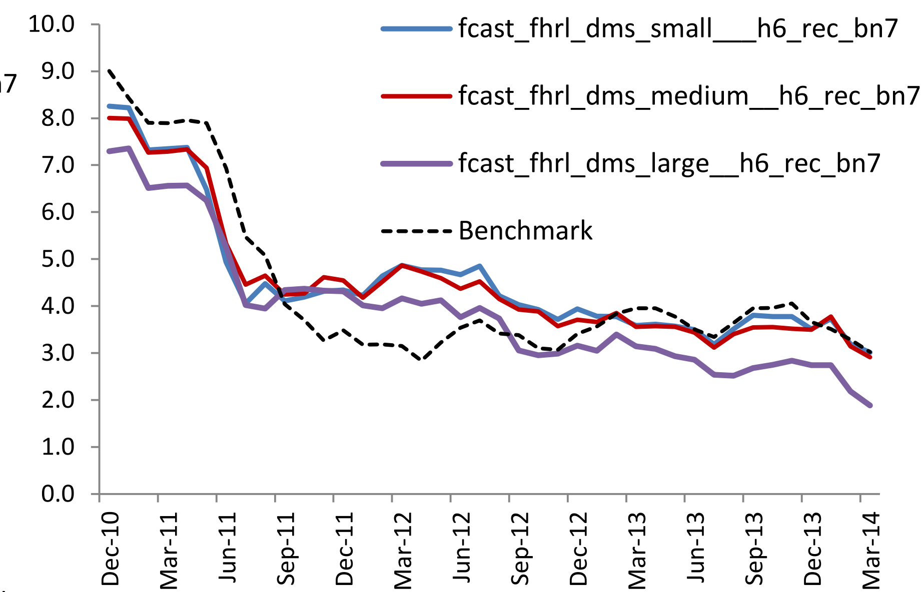
DI-AR-Lag model performs substantially worse, both from DI and from benchmark.

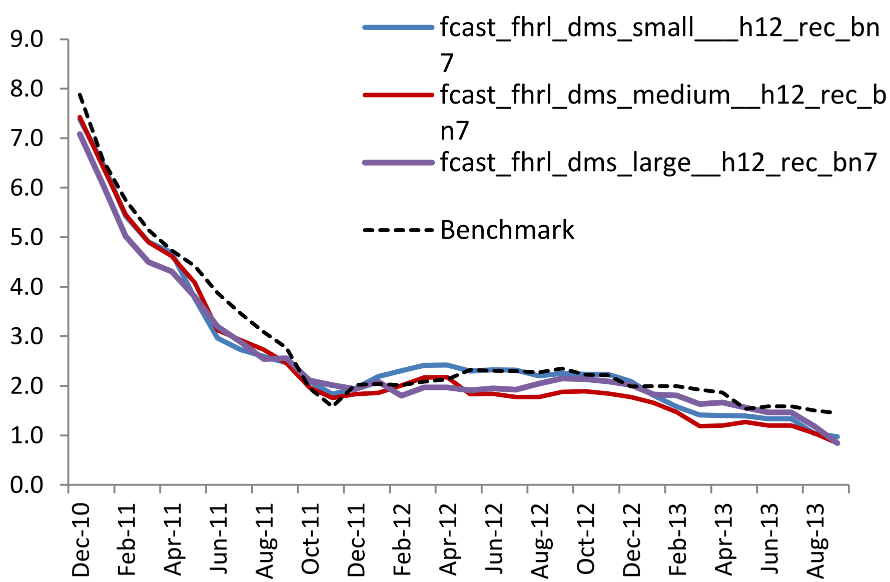
- On the other hand, for BIC3 where we work with a parsimonious model, **DI-AR-Lag** model performs relatively better.

Data Set Size

12 Month-Moving RMSEs for Different Data Sets given DMS and BIC3

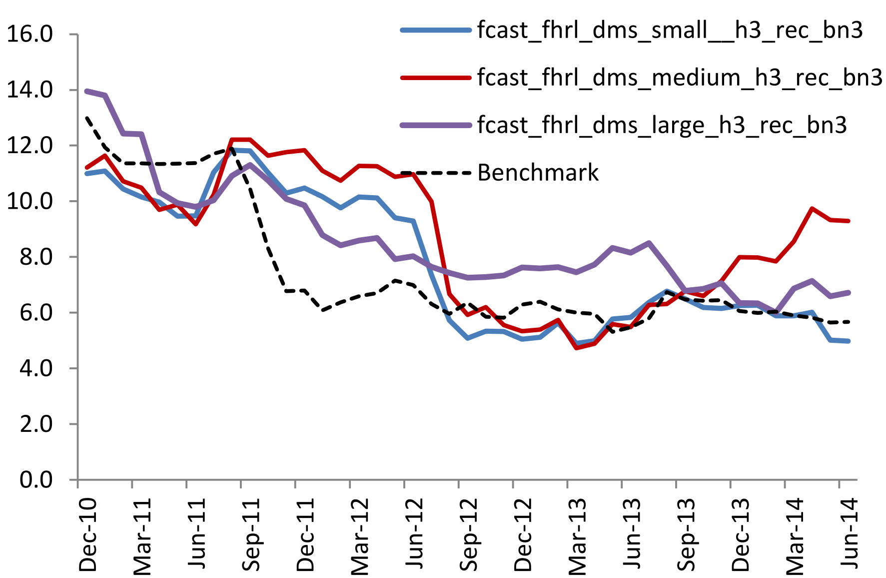


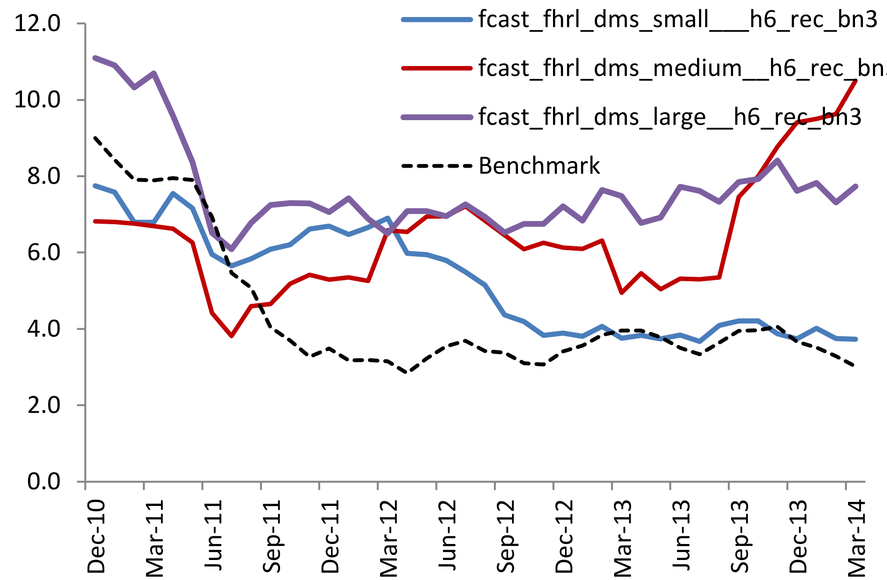


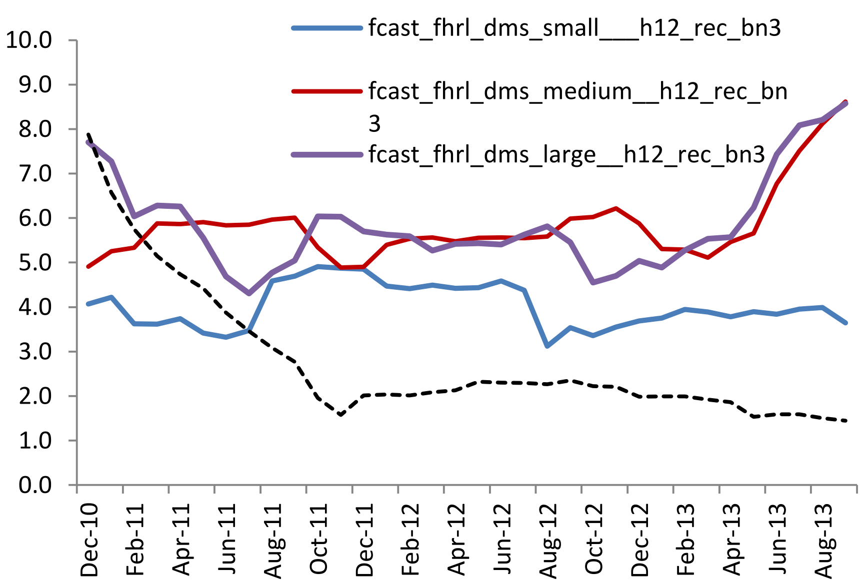


- For BIC3, large data set performs relatively and considerably better at short horizons while at a longer horizon medium performs a little better.

12 Month-Moving RMSEs for Different Data Sets given DMS and PC3







- For PC3, small data set performs relatively at all horizons while relative performance of large and small changes in different periods.

- **CPIH Results**

Best and Worst Performing Models for CPIH at h=3 for Two Episodes

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
IMS	FHRL	BIC3	M=H=8	Small/2	0.7004
IMS	FHRL	BIC3	M=H=4	Small/2	0.7009
IMS	FHRL	BIC3	M=H=16	Small/2	0.7012
IMS	PC	BIC3	-	Small/2	0.7017
IMS	PC	BIC3	-	Medium/2	0.7030
IMS	FHRL	BIC3	M=H=8	Small/2	0.7004

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS-F	FHRL	IC2	M=H=4	Small/4	0.74
AR IMS					0.74
DMS-F	FHRL	PC1	M=H=4	Small/4	0.78
DMS-F	FHRL	PC2	M=H=4	Small/4	0.78
DMS-F	FHRL	PC3	M=H=4	Small/4	0.78
DMS-F	FHRL	IC1	M=H=4	Small/4	0.78

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS-F	FHRL	PC1	M=H=16	Large/9	1.176
DMS-F	FHRL	PC3	M=H=16	Large/9	1.188
DMS-F	PC	PC2	-	Large/9	1.190
DMS	FHRL	PC3	M=H=4	Large/9	1.193
DMS-F	FHRL	IC1	M=H=8	Large/9	1.227
DMS-F	FHRL	IC2	M=H=8	Large/9	1.227

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS-F	PC	IC3	-	Large/9	1.20
DMS-F	FHRL	PC3	M=H=4	Medium/7	1.21
DMS-F	FHRL	IC3	M=H=4	Medium/7	1.21
DMS	FHRL	PC3	M=H=8	Medium/7	1.24
DMS	FHRL	IC3	M=H=8	Medium/7	1.24
DMS	FHRL	PC3	M=H=16	Medium/7	1.26

Best and Worst Performing Models for CPIH at h=6 for Two Episodes

Multistep Ahead Forecastin g Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
AR IMS					0.6344
IMS	FHRL	BIC3	M=H=8	Small/2	0.7983
IMS	FHRL	BIC3	M=H=4	Small/2	0.7986
IMS	PC	BIC3	-	Small/2	0.7990
IMS	FHRL	BIC3	M=H=16	Small/2	0.7991
IMS	PC	BIC3	-	Medium/2	0.8006

Multistep Ahead Forecasting Method	Factor Extracti on Metho d	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	PC	PC3	-	Large/9	1.212
DMS	PC	PC2	-	Large/9	1.213
DMS	PC	IC3	-	Large/9	1.218
DMS-F	FHRL	IC1	M=H=16	Large/9	1.239
DMS-F	FHRL	IC2	M=H=16	Large/9	1.239
DMS-F	FHRL	IC1	M=H=8	Large/9	1.240

Multistep Ahead Forecastin g Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	PC	IC2	-	Small/4	0.7312
DMS-F	FHRL	PC1	M=H=4	Small/4	0.7323
DMS-F	FHRL	PC2	M=H=4	Small/4	0.7323
DMS-F	FHRL	PC3	M=H=4	Small/4	0.7323
DMS-F	FHRL	IC1	M=H=4	Small/4	0.7323
DMS-F	FHRL	IC3	M=H=4	Small/4	0.7323

Multistep Ahead Forecating Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS-F	PC	IC3	-	Large/9	1.2823
DMS-F	PC	PC3	-	Large/9	1.2846
DMS	FHRL	PC3	M=H=8	Medium/7	1.2875
DMS	FHRL	IC3	M=H=8	Medium/7	1.2875
DMS-F	PC	PC3	-	Medium/7	1.3470
DMS-F	PC	IC3	-	Medium/7	1.3470

Best and Worst Performing Models for CPIH at h=9 for Two Episodes

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
AR IMS					0.7401
IMS	FHRL	BIC3	M=H=8	Small/2	0.8543
IMS	FHRL	BIC3	M=H=4	Small/2	0.8547
IMS	PC	BIC3	-	Small/2	0.8549
IMS	FHRL	BIC3	M=H=16	Small/2	0.8552
IMS	PC	BIC3	-	Medium/2	0.8570

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	PC1	M=H=4	Small/4	0.7835
DMS	FHRL	PC2	M=H=4	Small/4	0.7835
DMS	FHRL	PC3	M=H=4	Small/4	0.7835
DMS	FHRL	IC1	M=H=4	Small/4	0.7835
DMS	FHRL	IC3	M=H=4	Small/4	0.7835
DMS	FHRL	IC2	M=H=8	Small/4	0.7920

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	PC3	M=H=8	Small/4	1.284
DMS	FHRL	IC1	M=H=8	Small/4	1.284
DMS	FHRL	IC2	M=H=8	Small/4	1.284
DMS	FHRL	IC3	M=H=8	Small/4	1.284
DMS	PC	PC2	-	Large/9	1.296
DMS	FHRL	PC3	M=H=8	Medium/7	1.340

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS-F	PC	PC3	-	Large/9	1.1661
DMS-F	FHRL	PC3	M=H=8	Medium/7	1.2132
DMS-F	FHRL	IC3	M=H=8	Medium/7	1.2132
DMS-F	FHRL	PC3	M=H=16	Medium/7	1.2192
DMS-F	FHRL	IC3	M=H=16	Medium/7	1.2192
DMS-F	PC	PC3	-	Medium/7	1.2256

Best and Worst Performing Models for CPIH at h=12 for Two Episodes

Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode 1
AR IMS					0.8368
IMS	Bivariate	-	-	Small	0.9256
IMS	Bivariate	-	-	Large	0.9289
IMS	PC	BIC3	-	Small/2	0.9304
IMS	FHRL	BIC3	M=H=8	Small/2	0.9307
IMS	FHRL	BIC3	M=H=4	Small/2	0.9308

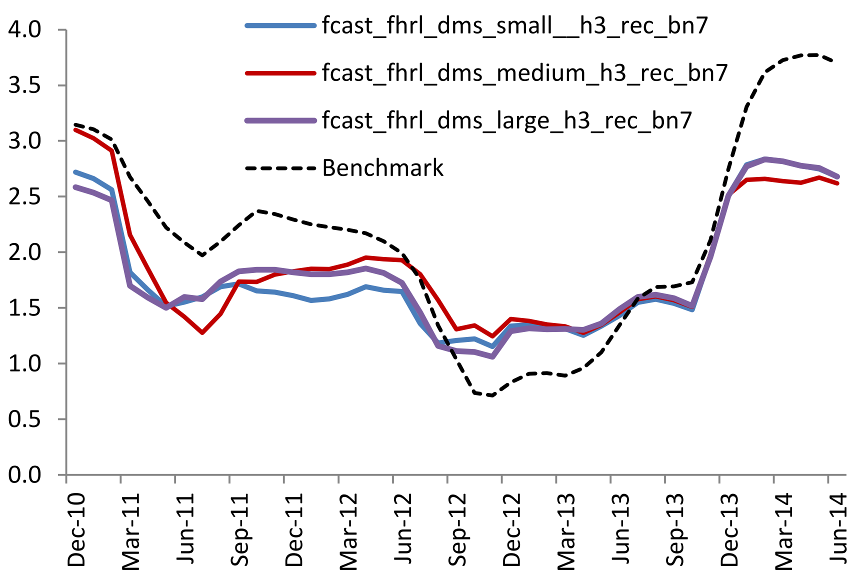
Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	IC2	M=H=4	Small/4	0.8870
DMS	FHRL	PC1	M=H=4	Small/4	0.8897
DMS	FHRL	PC2	M=H=4	Small/4	0.8897
DMS	FHRL	PC3	M=H=4	Small/4	0.8897
DMS	FHRL	IC1	M=H=4	Small/4	0.8897
DMS	FHRL	IC3	M=H=4	Small/4	0.8897

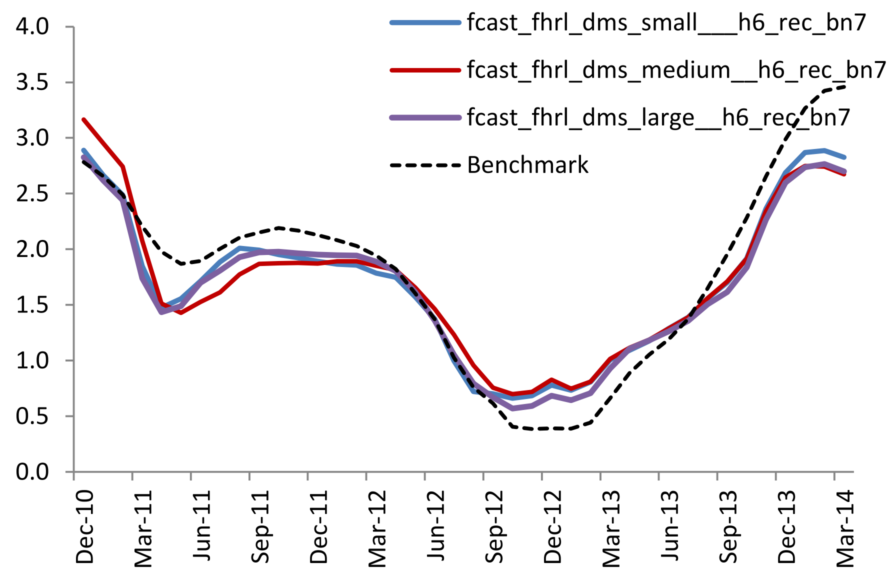
Multistep Ahead Forecassting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode1
DMS	FHRL	IC3	M=H=8	Large/9	1.881
DMS	FHRL	PC1	M=H=8	Medium/7	1.884
DMS	FHRL	PC2	M=H=8	Medium/7	1.885
DMS	FHRL	PC3	M=H=8	Medium/7	1.897
DMS	FHRL	IC3	M=H=8	Medium/7	1.897
DMS	FHRL	PC1	M=H=4	Large/9	1.912

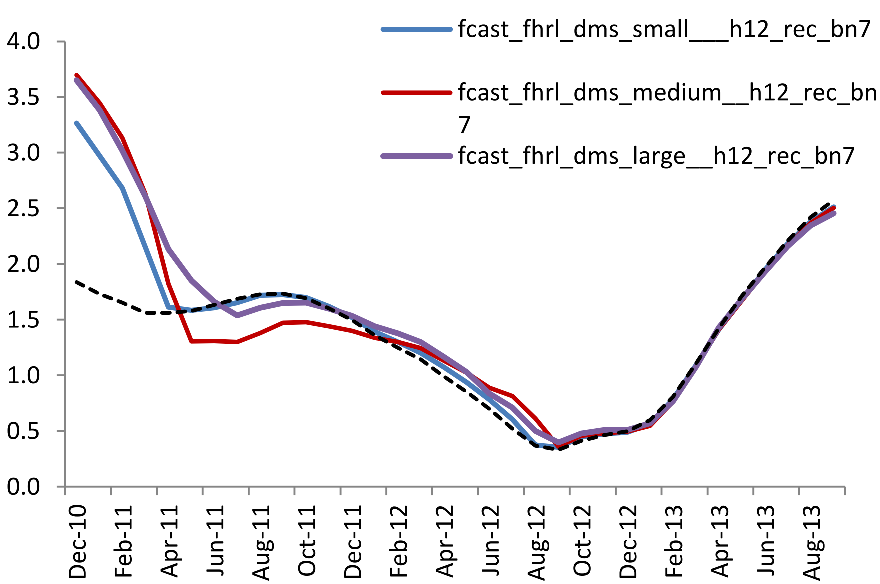
Multistep Ahead Forecasting Method	Factor Extraction Method	Number of Static Factor Selection Method	M and N in Bai and Ng (2005) for Number of Dynamic Factor	Data Set Size and Maximum Number of Factors	Episode2
DMS	PC	IC3	-	Large/9	1.1255
DMS-F	FHRL	PC3	M=H=8	Medium/7	1.1315
DMS-F	FHRL	IC3	M=H=8	Medium/7	1.1315
DMS-F	FHRL	PC3	M=H=16	Medium/7	1.1439
DMS-F	FHRL	IC3	M=H=16	Medium/7	1.1439
DMS-F	PC	PC3	-	Medium/7	1.1474

Data Set Size

12 Month-Moving RMSEs for Different Data Sets given DMS and BIC3

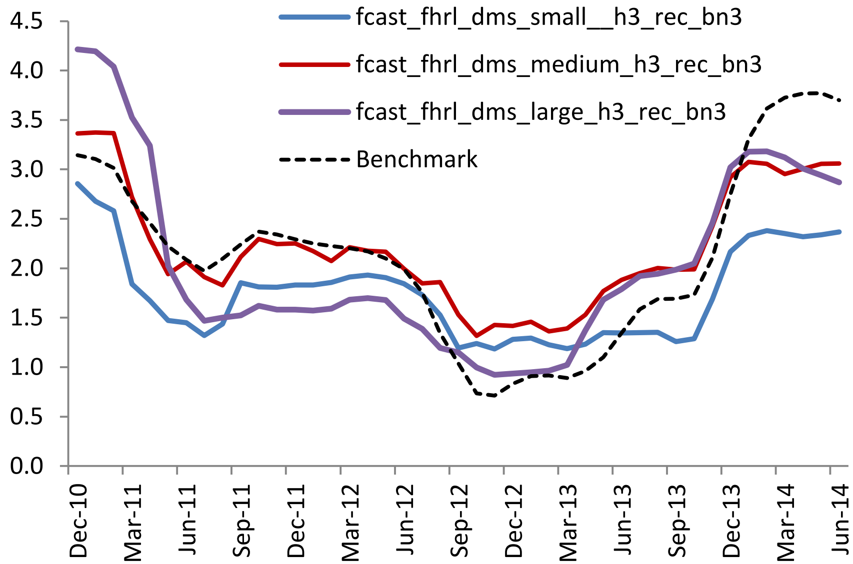


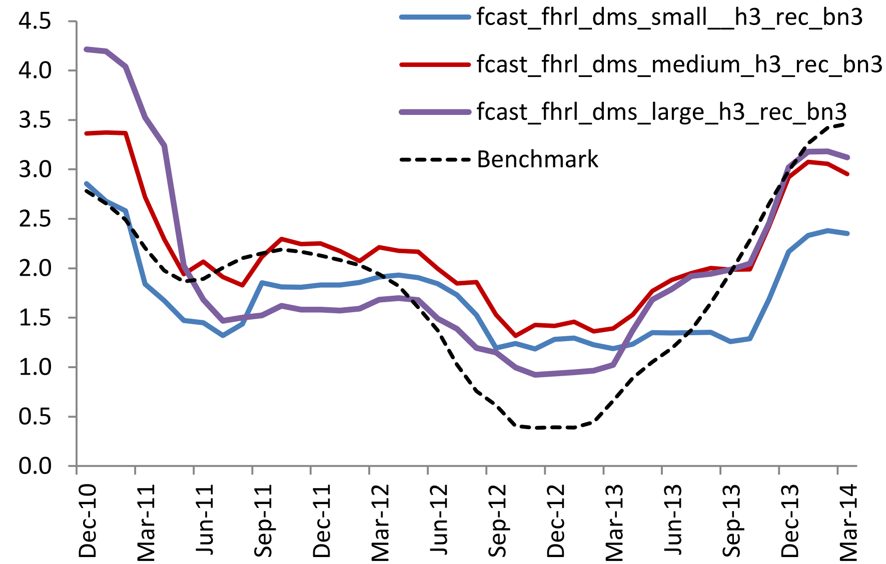


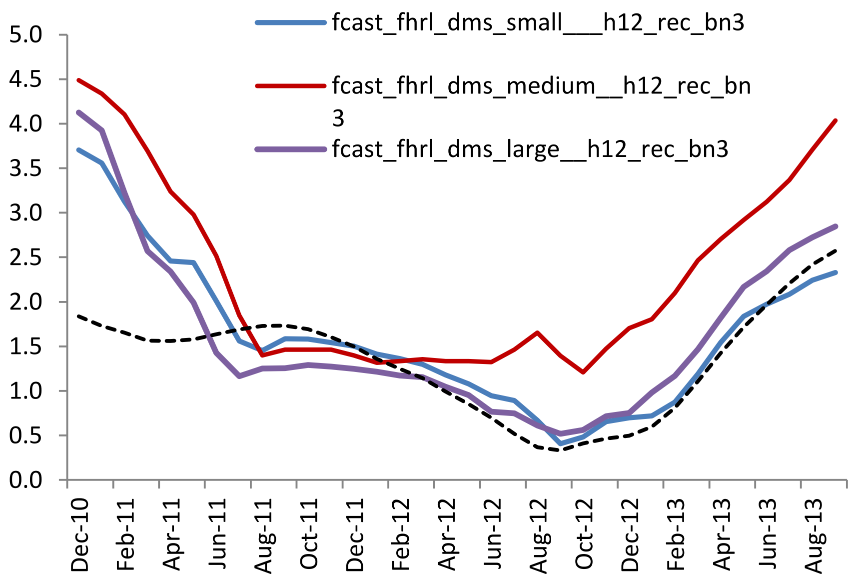


- Data set size does not play a big role on the forecast performance given BIC3.

12 Month-Moving RMSEs for Different Data Sets given DMS and PC3



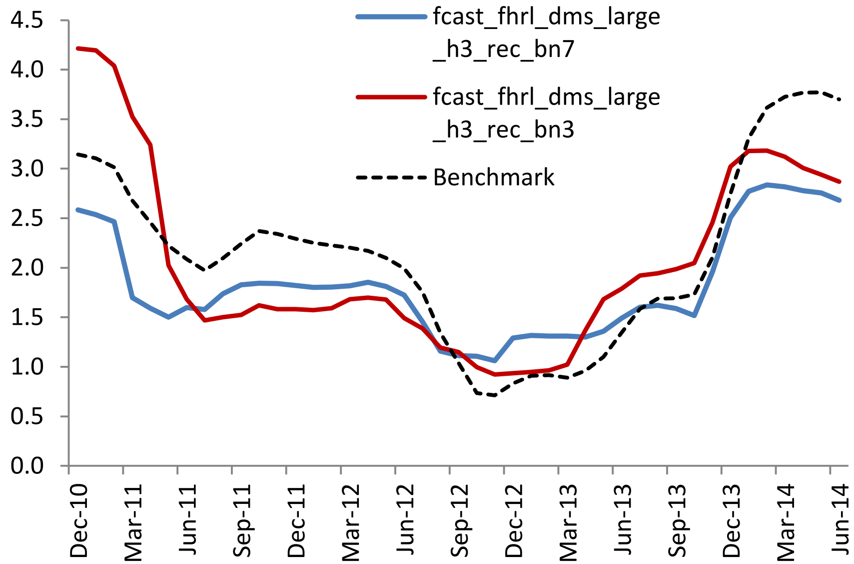


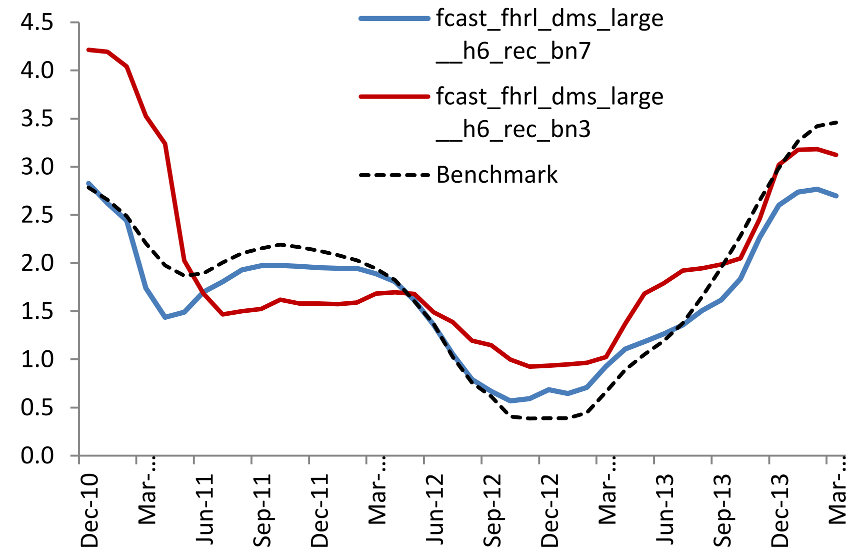


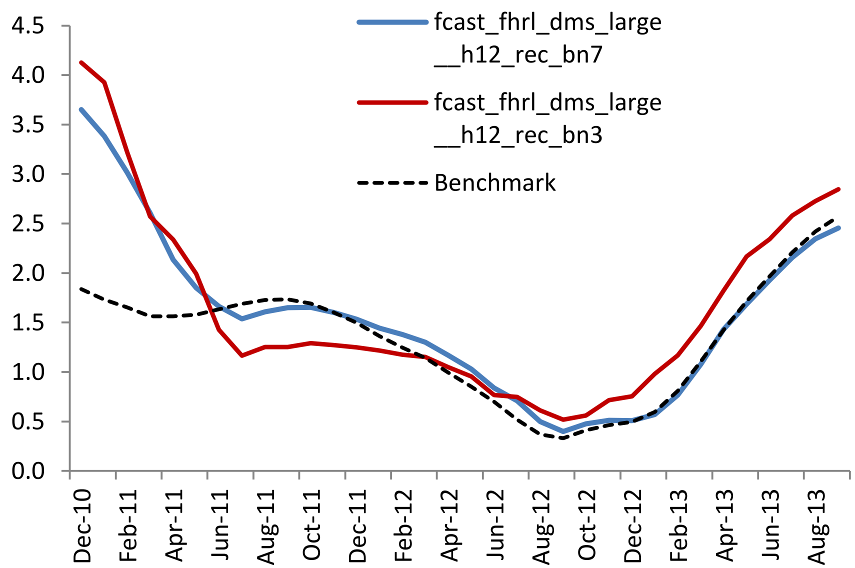
❖ Data set size plays a significant role on the forecast performance given PC3.

- **Number of Factors**

Rolling 12 Month RMSEs for Different Specifications, BIC3 vs PC3 given FHLR and Large data set



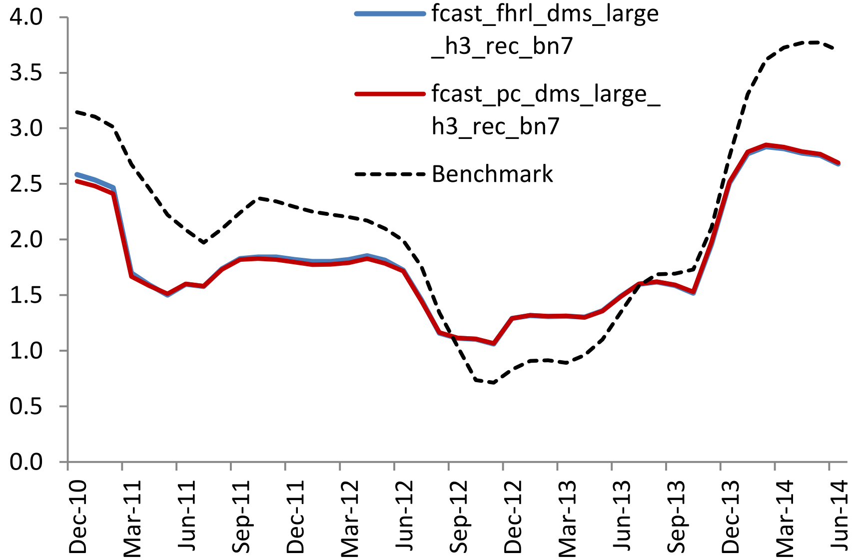


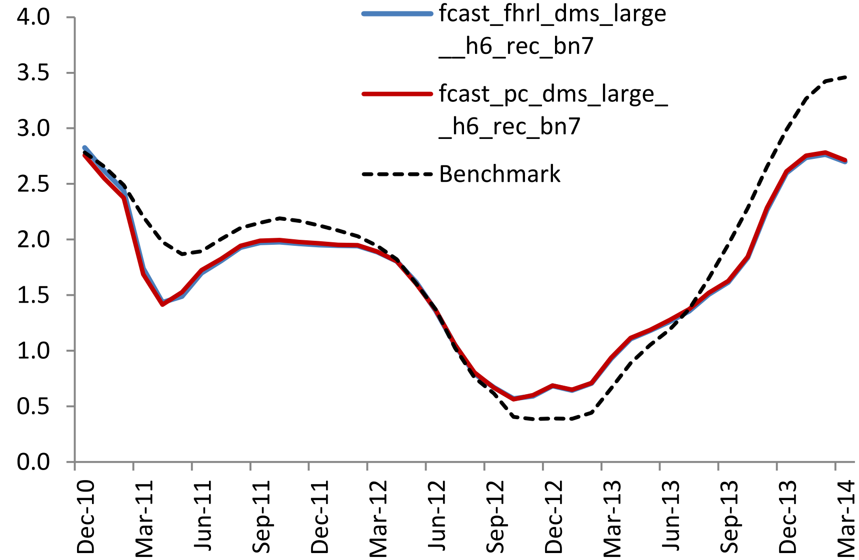


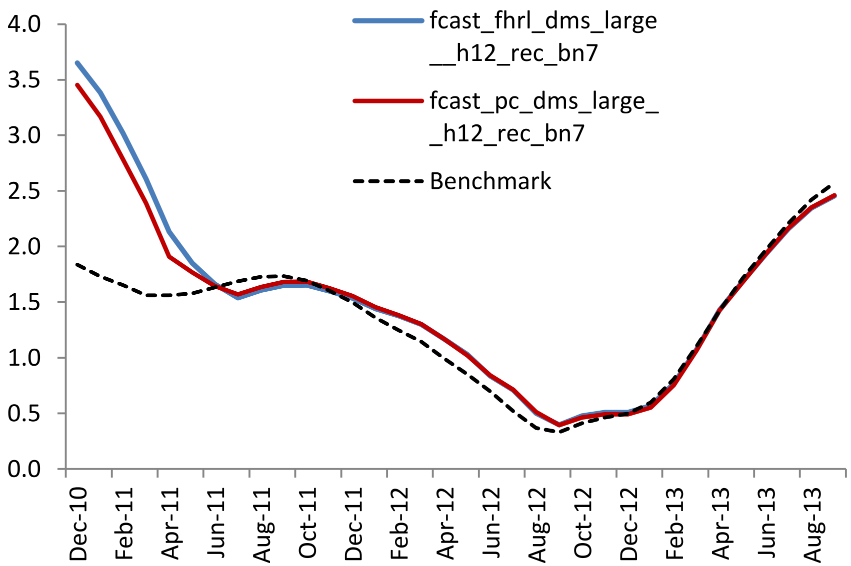
- Increasing the parameters in the model by increasing the number of factors affect differently depending on the forecast horizon.

FHRL vs PC

Rolling 12 Month RMSEs for Different Specifications, PC vs FHLR given BIC3 and Large data set

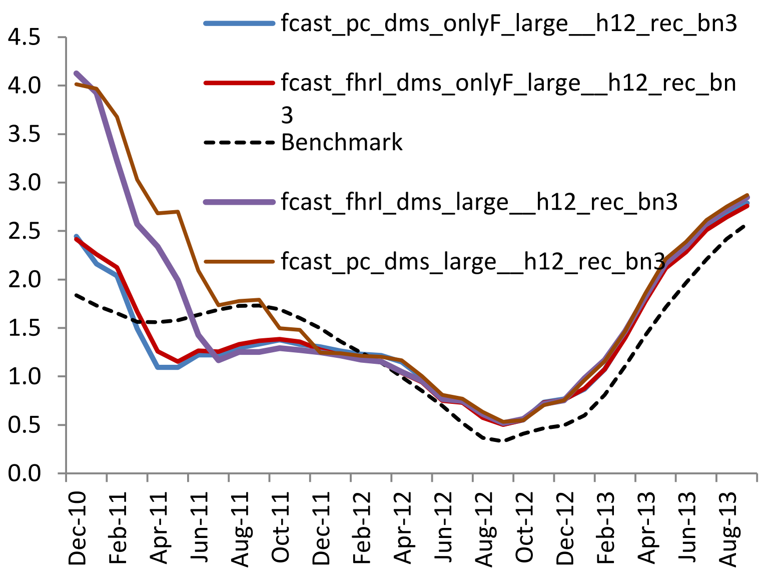


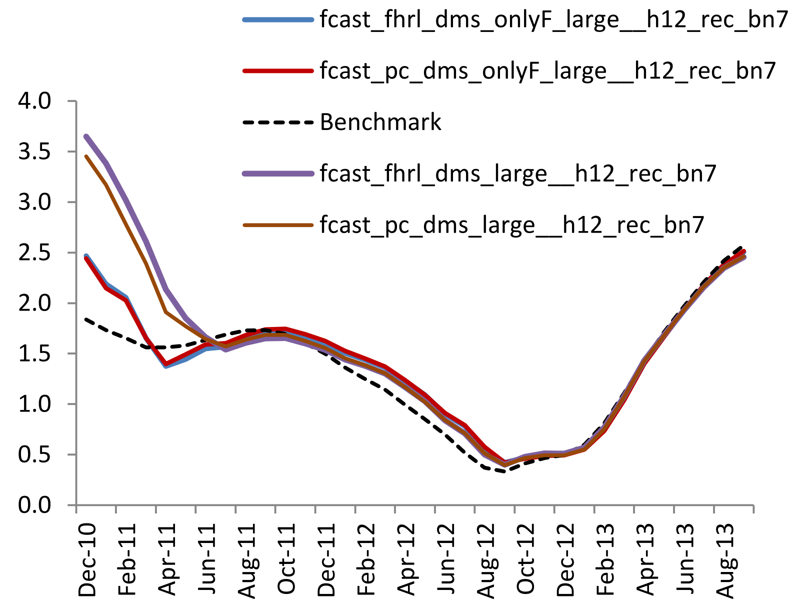




- Increasing the parameters in the model by increasing the number of factors affect differently depending on the forecast horizon.

Rolling 12 Month RMSEs for Different Specifications, PC vs FHLR for PC3 vs BIC3 given h12 and Large data set



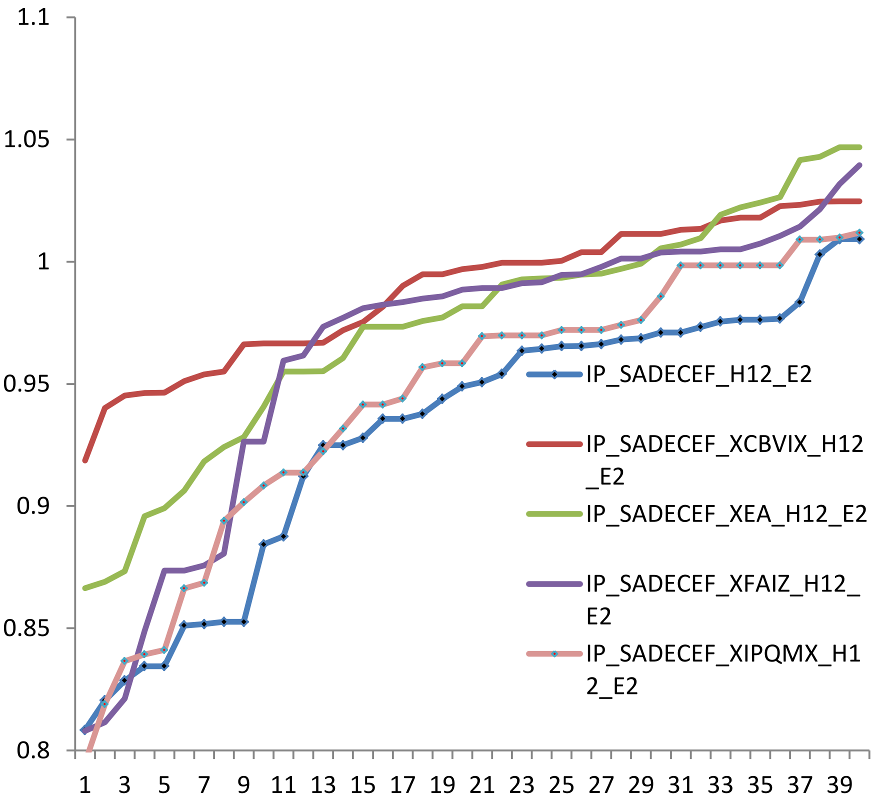


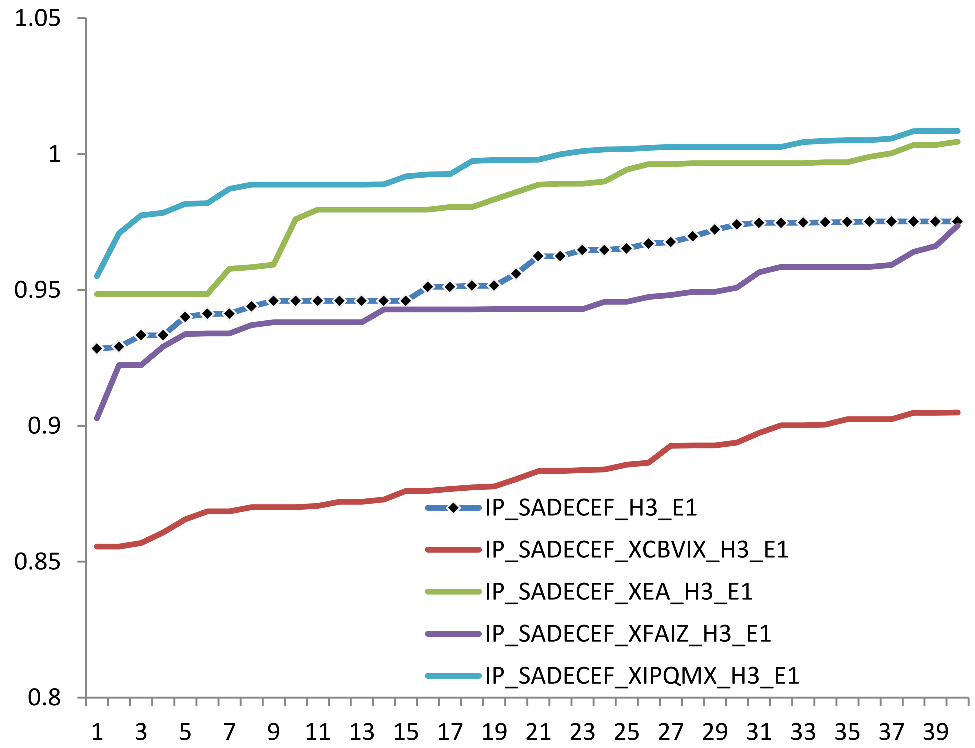
- Ongoing work: Effect of different blocks

Literature also considers excuding some blocks to see that blocks predictive content: Forecasting national activity using lots of international predictors: an application to New Zealand, 2011, Eickmeier and Ng, International Journal of Forecasting.

- Abstract
- We assess the marginal predictive content of a large international dataset for forecasting GDP in New Zealand, an archetypal small open economy. We apply “data-rich” factor and shrinkage methods to efficiently handle hundreds of predictor series from many countries. The methods covered are principal components, targeted predictors, weighted principal components, partial least squares, elastic net and ridge regression. We find that exploiting a large international dataset can improve forecasts relative to data-rich approaches based on a large national dataset only, and also relative to more traditional approaches based on small datasets. This is in spite of New Zealand’s business and consumer confidence and expectations data capturing a substantial proportion of the predictive information in the international data. The largest forecasting accuracy gains from including international predictors are at longer forecast horizons. The forecasting performance achievable with the data-rich methods differs widely, with shrinkage methods and partial least squares performing best in handling the international data.

Preliminary results for excluding blocks, RMSEs relative to Benchmark





*Views expressed in this presentation are those of the author and do not necessarily represent the official views of the Central Bank of the Republic of Turkey

Comparing Alternative Factor Models for Forecasting: Case of Turkey

June 1, 2015

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